which is not row sum dominant for all $c > 1,\ d < 1 + \sqrt{2}$, but S is not row sum dominant and is not a nonsingular M-matrix. Define a set B_{cd}:

$$B_{cd} = \Big\{(c,\ d)|c > 1,\ 0 < d < 1 + \sqrt{2},\ 2cd + \Big(2 - \sqrt{2}\Big)(c + d) < 4\sqrt{2}\Big\}.$$

It is easily seen that $B_{cd} \neq \emptyset$ and the results of [4], [11], [18] cannot be used as $(c,\ d) \in B_{cd}$. However, take $(c,\ d) = (1.1,\ 0.51) \in B_{cd}$, $r = 0$, we can easily check that the conditions in theorem 1 or corollary 1 given herein are satisfied (here, the main theorem in [17] cannot also be applied at this time). Hence system (1) has a unique and globally asymptotically stable equilibrium point.

Our results provide two parameter $a,\ r$ to appropriately compensate for the tradeoff between matrix definite condition on feedback matrix and the norm inequality condition on delayed feedback matrix. The less restrictive norm condition on the delayed feedback matrix is with respect to $a,\ r$. Therefore, this condition herein is less restrictive than that given in the earlier references.

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Dynamic Behavior of Dynamic Translinear Circuits: The Linear Time-Varying Approximation

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***Abstract*—Dynamic translinear circuits explore the exponential relation of transistors as a primitive for the synthesis of electronic circuits. In this letter, the linear time-varying approximation is applied to describe the dynamic behavior of a second-order dynamic-translinear oscillator. The Floquet exponents are calculated by the earlier introduced dynamic eigenvalues.**

***Index Terms*—Dynamic translinear circuits, log-domain, nonlinear dynamic circuits, linear time-varying approximation, stability, floquet exponents.**

I. INTRODUCTION

In the synthesis of *dynamic-translinear circuits* (DTL circuits) the exponential input-output relation of the bipolar transistor or of the MOS transistor in its weak inversion regime, is used as a primitive. These circuits are based on the so-called dynamic translinear principle [1], a generalization of the well-known static translinear principle [2]. Whereas conventional TL circuits can be used to implement various linear and nonlinear *static* transfer functions, DTL circuits can implement a wide variety of *dynamic* functions, described by differential equations (DEs). In this way, both linear DEs, e.g., filters [3]–[6], and nonlinear DEs, e.g., oscillators [7], [8] can be realized.

The *linear time-varying* (LTV) *approximation* is a general method to describe the local dynamics of nonlinear systems [9]. It is a consistent generalization of the LTI small-signal approximation, in which the eigenvalue and pole concept are generalized by means of the so-called *dynamic* eigenvalue [10], [11].

In this letter, we apply the LTV approximation to describe the dynamic behavior of a DTL oscillator. In Section II the dynamic translinear principle is reviewed. Then, the LTV approximation is

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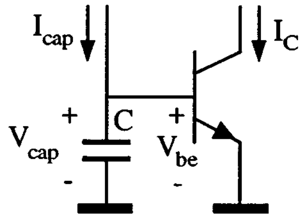

Fig. 1. Basic structure of DTL circuits.

introduced in Section III and subsequently applied to a DTL oscillator in Section IV. Conclusions are drawn in Section V.

II. The Dynamic-Translinear Principle

The static translinear principle can be extended in order to implement differential equations, by adding capacitors in the TL loop [1]. The DTL principle is explained referring to Fig. 1.

Using a formulation based on currents (ignoring the base current), this circuit is described in terms of the collector current I_c and the capacitor current $I_{\rm cap}$, flowing through the capacitor C. Current $I_{\rm cap}$ in terms of products of currents is given by [1]

$$C \cdot V_T \dot{I}_c = I_{\rm cap} \cdot I_c \tag{1}$$

where the dot operator indicates differentiation with respect to the time and V_T is the thermal voltage. Equation (1) reflects the DTL principle: *a time derivative can be mapped onto a product of currents*, allowing us to map a DE onto a multivariable polynomial of currents.

III. Linear Time-Varying Approach

In this section we review the LTV approach [9]. A nonlinear circuit can be described by the following state-space equation:

$$\dot{\mathbf{w}}(t) = \mathbf{f}(\mathbf{w}(t), \mathbf{s}(t)). \tag{2}$$

Here, $\mathbf{w}$ and $\mathbf{s}$ are the state variable and the input vector, respectively, while $\mathbf{f}$ represents a nonlinear vector function. The signal-dependent bias trajectory or *dynamic bias trajectory* $\mathbf{w}_b = \mathbf{w}_b(t)$is given by the nonlinear and time-dependent solution of

$$\dot{\mathbf{w}}_b = \mathbf{f}(\mathbf{w}_b, \mathbf{s}_b) \tag{3}$$

with $\mathbf{s}_b$ denoting the external sources. The variational equation or LTV small-signal model describes the dynamic behavior of the system by considering small variations around the bias trajectory $\mathbf{w}_b$ and the external signal $\mathbf{s}_b$, respectively. That is, we consider any state and any external signal of the system to be composed of the sum

$$\mathbf{w} = \mathbf{w}_b + \mathbf{x} \text{ and } \mathbf{s} = \mathbf{s}_b + \mathbf{e} \tag{4}$$

where $\mathbf{x}$ and $\mathbf{e}$ are relatively small variations in the state and in the external signal, respectively. If we linearise the state-space (2) around $\mathbf{w}_b$ and $\mathbf{s}_b$, the following variational equation is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{e} \tag{5}$$

in which the system matrix $\mathbf{A}$ is the Jacobian of $\mathbf{f}$ with respect $\mathbf{w}$, and $\mathbf{B}$ the Jacobian of $\mathbf{f}$ with respect to $\mathbf{s}$, both evaluated along the bias trajectory.

Subsequently, we study the dynamic behavior using the relevant part of the variational Eq. (5), i.e., $\mathbf{e}$ is assumed to be zero

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}. \tag{6}$$

The dynamic behavior of the system for given inputs can be described by time-varying modes of the homogeneous variational equation (6). Thus, a solution of (6) can be written as a linear combination of modes $\mathbf{x}_i(t)$ [10]–[13]

$$\mathbf{x}(t) = \sum_i \mathbf{x}_i(t) = \sum_i \mathbf{u}_i(t)\exp[\gamma_i(t)] \tag{7}$$

where $\mathbf{u}_i(t)$ denotes a time-dependent amplitude, while the time-dependent phase $\gamma_i(t)$ is written as

$$\gamma_i(t) = \int_0^t \lambda_i(\tau)d\tau \Leftrightarrow \dot{\gamma}_i(t) = \lambda_i(t) \tag{8}$$

in which $\lambda_i(t)$ is a time-dependent frequency. Substitution of (7) in (6) yields the following *dynamic* eigenvalue problem:

$$(\mathbf{A} - \lambda_i\mathbf{I})\mathbf{u}_i = \dot{\mathbf{u}}_i \tag{9}$$

where $\mathbf{I}$ is the unity matrix. Note that the classical static eigenvalue problem results if $\mathbf{u}_i$ is time independent. In the context of Eq. (9) the physical quantities $\mathbf{u}_i(t)$ and $\lambda_i(t)$ are called a *dynamic* eigenvector and a *dynamic* eigenvalue, respectievly.

Next, the following transformation is applied:

$$\mathbf{x} = \mathbf{L}(t)\mathbf{y} \tag{10}$$

by which (6) goes into the new time-varying system

$$\dot{\mathbf{y}} = (\mathbf{L}^{-1}\mathbf{A}_x\mathbf{L} - \mathbf{L}^{-1}\dot{\mathbf{L}})\mathbf{y} = \mathbf{C}(t)\mathbf{y}. \tag{11}$$

On account of (7) and (10), the solution $\mathbf{y}$ reads

$$\mathbf{y}(t) = \sum_i \mathbf{L}^{-1}(t)\mathbf{u}_i(t)\exp[\gamma_i(t)] = \sum_i \mathbf{v}_i(t)\exp[\gamma_i(t)] \tag{12}$$

where $\mathbf{v}_i(t)$ is a dynamic eigenvector of the new system (11). Thus, transformation (10) preserves the dynamic eigenvalues as defined in (8). For this reason, the system matrices $\mathbf{A}$ and $\mathbf{C}$ are called *dynamic* similar [10]. In the next section, the dynamic similarity transform $\mathbf{L}$ is chosen such that $\mathbf{C}$ becomes an upper-triangular system matrix. Then, the dynamic eigenvalues are its main diagonal elements [11], [12].

Finally, for periodic systems the Floquet exponents, given by

$$\beta_i = \frac{1}{T}\gamma_i(T) = \frac{1}{T}\int_0^T \lambda_i(\tau)d\tau \tag{13}$$

are a measure for stability.

IV. Analysis of a DTL Oscillator

In this section, we apply the LTV approximation to the analysis of the dynamic behavior of the DTL oscillator introduced in [8]. We show that the dynamic eigenvalues of a second-order system can be found by solving a scalar Riccati differential equation.

A. The Linear Time-Varying Approach for Second-Order Systems

We first give a systematic method to find the dynamic eigenvalues of a second-order variational equation by applying a suitable dynamic similarity transformation [13]. The problem is reduced to solving a scalar Riccati differential equation. Then, we introduce a variable transformation in order to obtain a linear equation, which is more suitable for numerical calculation.

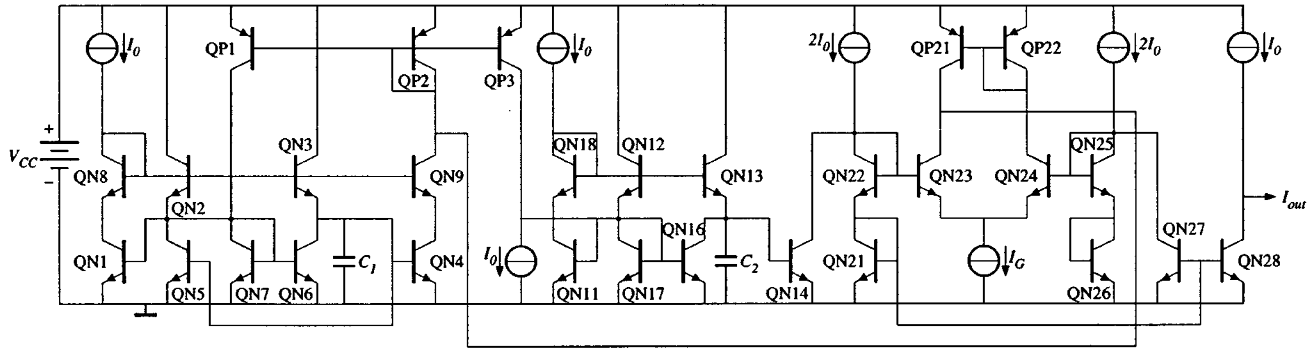

Fig. 2. Circuit implementation of a DTL oscillator [8].

1) The Dynamic Eigenvalues: We start with the second-order variational state-equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) \Leftrightarrow \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{14}$$

In order to find the dynamic eigenvalues, the dynamic eigenvalue problem (9) has to be solved. The following dynamic similarity transformation $\mathbf{L}(t)$ is applied to $\mathbf{x}(t)$:

$$\mathbf{x}(t) = \mathbf{L}(t)\mathbf{y}(t) \Leftrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ l(t) & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{15}$$

in which $l(t)$ is a time-dependent variable. The transformed variational equation is given by

$$\begin{aligned} \dot{\mathbf{y}}(t) &= (\mathbf{L}^{-1}\mathbf{A}_x\mathbf{L} - \mathbf{L}^{-1}\dot{\mathbf{L}})\mathbf{y}(t) = \mathbf{C}(t)\mathbf{y}(t) \\ &\Leftrightarrow \frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \lambda_1(t) & a_{12} \\ 0 & \lambda_2(t) \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \end{aligned} \tag{16}$$

provided that $l(t)$ in (15) is any solution of the scalar Riccati differential equation

$$\dot{l} = -a_{12}l^2 - (a_{11} - a_{22})l + a_{21} \tag{17}$$

and where the dynamic eigenvalues $\lambda_1(t)$ and $\lambda_2(t)$ are given by

$$\begin{aligned} \lambda_1(t) &= a_{11}(t) + l(t)a_{12}(t) \text{ and} \\ \lambda_2(t) &= -l(t)a_{12}(t) + a_{22}(t). \end{aligned} \tag{18}$$

2) Transformation of the Riccati Differential Equation: If we deal with a time-varying variational equation, the Riccati equation (17) should be solved. In general, this equation has no analytical solution. Even numerical calculation is difficult as singularities (finite escape times) may be involved. In order to facilitate the calculations, we introduce the new variable $u(t)$ as

$$u(t) = \exp\left[\int_0^t a_{12}(t)l(t)dt\right] \Leftrightarrow a_{12}(t)l(t) = \frac{\dot{u}(t)}{u(t)}. \tag{19}$$

By applying (19), the Riccati (17) goes into the following second-order linear time-varying DE:

$$\begin{aligned} -a_{12}(t)\ddot{u} + [\dot{a}_{12}(t) - a_{12}(t)(a_{11}(t) - a_{22}(t))]\,\dot{u} \\ + a_{12}(t)^2 a_{21}(t)u = 0 \end{aligned} \tag{20}$$

which on its turn is written in the state-space formulation

$$\begin{cases} \dot{u} = q \\ \dot{q} = \left[\dfrac{\dot{a}_{12}(t) - a_{12}(t)(a_{11}(t) - a_{22}(t))}{a_{12}(t)}\right] q + a_{12}(t)a_{21}(t)u \end{cases} \tag{21}$$

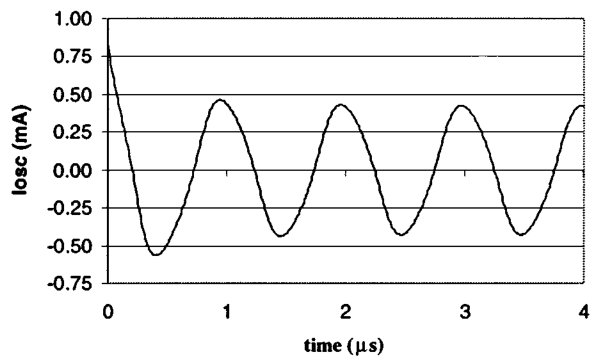

Fig. 3. Oscillator output as function of time for $C_1 = C_2 = 5$ nF, $I_0 = 817$ uA, $G = 1.2$.

The new state-variables u and q can be added to the original nonlinear system (3): a fourth-order-state-space system results. The solutions of this system are used to find both the dynamic bias trajectory and the solutions of the two state-space variables u and q, respectively. The solution of the Riccati equation $l(t)$ is given by

$$l(t) = \frac{1}{a_{12}(t)}\frac{q}{u} \tag{22}$$

while the dynamic eigenvalues follow from (18). Since for the example to be discussed the dynamic eigenvalues and eigenvectors are periodic, we can use (13) to determine the Floquet exponents. In order to calculate these exponents directly from the solution of the transformed Riccati equation, we rewrite (19) as

$$\int_0^t a_{12}(\tau)l(\tau)d\tau = \ln[u(t)]. \tag{23}$$

Then, substitution of (23) in (18) and using (13) yields

$$\begin{aligned} \beta_1 &= \frac{1}{T}\ln(u)\,\Big|_{t'}^{t'+T} + \frac{1}{T}\int_0^T a_{11}(t)dt \\ \beta_2 &= -\frac{1}{T}\ln(u)\,\Big|_{t'}^{t'+T} + \frac{1}{T}\int_0^T a_{22}(t)dt \end{aligned} \tag{24}$$

where β_1 and β_2 are the two Floquet exponents of the second-order system.

B. DTL Second-Order Oscillator

The complete design of the second-order DTL oscillator is described in [8]. The circuit diagram is depicted in Fig. 2. Here, we will use the nonlinear differential equation describing this DTL oscillator [8]

$$C^2V_T^2\ddot{I}_{\text{osc}} + CV_TI_0 2\left\{1 - GI_0^2\left[\frac{I_0^2 - I_{\text{osc}}^2}{(I_0^2 + I_{\text{osc}}^2)^2}\right]\right\} \times \dot{I}_{\text{osc}} + I_0^2 I_{\text{osc}} = 0 \tag{25}$$

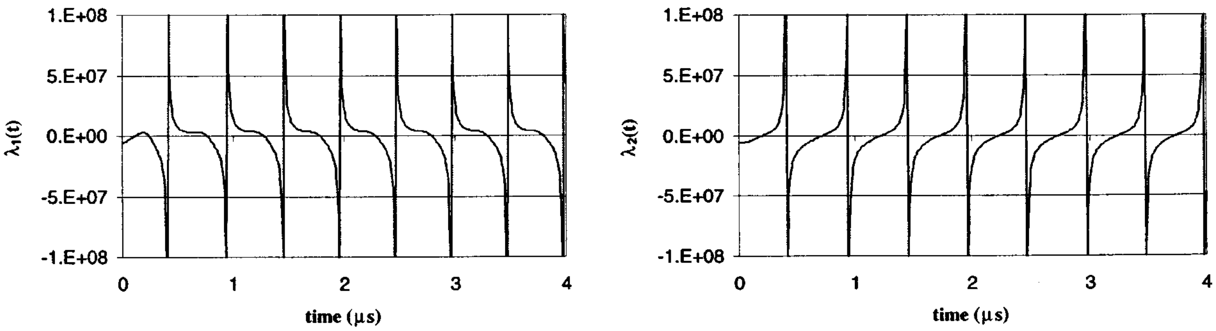

Fig. 4. Real part of the dynamic eigenvalue $\lambda_1(t)$ and $\lambda_2(t)$ as function of time.

in which $C = C_1 = C_2, V_T$ is the thermal voltage, $I_{\rm osc}$ is the output current of the oscillator ($I_{\rm out}$ in Fig. 2), I_0 a bias current and $G = I_G/I_0$ (I_G is the tail current of the differential pair QN23-QN24 in Fig. 2).

The oscillation frequency and amplitude are, respectively, given by [8]

$$\omega = \frac{I_0}{V_T C} \text{ and } \hat{I}_{\rm osc} = I_0\sqrt{G-1}. \tag{26}$$

C. The LTV Approach Applied to the Second-Order DTL Oscillator

We now use the LTV approach to describe the dynamic behavior of the DTL oscillator.

1) The State-Space Description: Rewriting (25), the oscillator can be described as a state-space system like (2) with $\mathbf{s}(t) = 0$ (w_1 and w_2 are the state-space variables)

$$\begin{cases} \dot{w}_1 = w_2 - \omega w_1 \\ \dot{w}_2 = 2GI_0^2\omega\left[\dfrac{I_0^2 - w_1^2}{(I_0^2+w_1^2)^2}\right](w_2 - \omega w_1) - \omega w_2 \end{cases}. \tag{27}$$

2) The Dynamic Bias Trajectory: We consider the state variables to consist of the sum

$$w_1 = w_{1b} + x_1 \text{ and } w_2 = w_{2b} + x_2 \tag{28}$$

where w_{1b} and w_{2b} are the bias-trajectories of the state-variables and x_1, x_2 are relatively small variations on w_{1b} and w_{2b}, respectively. The dynamic bias trajectory is given by the solution of (27) for $x_1 = x_2 = 0$.

3) The Variational Equation: The variational equation (6) is obtained from (27) as

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ a_{11}(t) &= \frac{\partial f_1}{\partial w_1}(w_{1b}, w_{2b}) = -\omega \\ a_{12}(t) &= \frac{\partial f_1}{\partial w_2}(w_{1b}, w_{2b}) = 1 \end{aligned} \tag{29}$$

$$\begin{aligned} a_{21}(t) &= \frac{\partial f_2}{\partial w_1}(w_{1b}, w_{2b}) \\ &= 2G\omega^2 I_0^2\left[\frac{-w_{1b}^4 + 6I_0^2 w_{1b}^2 - I_0^4}{(w_{1b}^2+I_0^2)^3}\right] \\ &\quad + 2G\omega I_0^2 w_{2b}\left[\frac{2w_{1b}^3 - 6I_0^2 w_{1b}}{(w_{1b}^2+I_0^2)^3}\right] \end{aligned} \tag{30}$$

$$\begin{aligned} a_{22}(t) &= \frac{\partial f_2}{\partial w_2}(w_{1b}, w_{2b}) \\ &= 2G\omega I_0^2\left[\frac{I_0^2 - w_{1b}^2}{(w_{1b}^2+I_0^2)^2}\right] - \omega. \end{aligned} \tag{31}$$

Note that the elements a_{11}, a_{12}, a_{21} and a_{22} are time-dependent since they are function of the dynamic bias trajectory (w_{1b}, w_{2b}).

4) The Dynamic Eigenvalues: To calculate the dynamic eigenvalues, the Riccati equation must be solved. Substitution of a_{11}, a_{12}, a_{21}, and a_{22} in (17) yields

$$\begin{aligned} \dot{l}(t) = &-l(t)^2 - 2G\omega I_0^2\left[\frac{I_0^2 - w_{1b}^2}{(w_{1b}^2+I_0^2)^2}\right]\cdot l(t) \\ &+ 2G\omega^2 I_0^2\left[\frac{-w_{1b}^4 + 6I_0^2 w_{1b}^2 - I_0^4}{(w_{1b}^2+I_0^2)^3}\right] \\ &+ 2G\omega I_0^2 w_{2b}\left[\frac{2w_{1b}^3 - 6I_0^2 w_{1b}}{(w_{1b}^2+I_0^2)^3}\right]. \end{aligned} \tag{32}$$

This is a time-varying quadratic differential equation in the unknown $l = l(t)$. Its solutions is found via transformation (19). Substitution of (22) and (29)–(31) in (18) gives the following dynamic eigenvalues:

$$\begin{aligned} \lambda_1(t) &= \frac{q}{u} - \omega \quad \text{and} \\ \lambda_2(t) &= -\frac{q}{u} + 2G\omega I_0^2\left[\frac{I_0^2 - w_{1b}^2}{(I_0^2+w_{1b}^2)^2}\right] - \omega \end{aligned} \tag{33}$$

where u and q are the solutions of the state-space system (21).

5) The Floquet Exponents: The Floquet exponents are given by substituting (29)–(31) in (24)

$$\begin{aligned} \beta_1 = \left[\frac{1}{T}\ln(u)\,\Big|_{t'}^{t'+T}\right] - \omega \text{ and } \beta_2 = -\left[\frac{1}{T}\ln(u)\,\Big|_{t'}^{t'+T}\right] \\ + \frac{1}{T}\int_0^T 2G\omega I_0^2\left[\frac{I_0^2 - w_{1b}^2}{(w_{1b}^2+I_0^2)^2}\right]dt - \omega. \end{aligned} \tag{34}$$

Any oscillator is characterized by a constant oscillation amplitude in steady state. As a consequence, one Floquet exponent should equal zero and the other Floquet exponent should have a negative real part. In the following paragraph this statement is checked by a numerical example.

6) Numerical Example: Suppose an oscillating frequency of 1 MHz is specified. We chose the capacitors to be $C = 5$ nF. It follows from (26) that $I_0 = 817$ μA. The oscillator output signal (w_{1b}) is plotted for $G = 1.2$ in Fig. 3. Notice that the estimation of the amplitude according to (26) ($\hat{I}_{\rm osc} = 365$ μA) corresponds to the simulated value. The dynamic eigenvalues follow from (33). The real parts are plotted in Fig. 4. The imaginary parts of both dynamic eigenvalues are equal to zero. Notice that both dynamic eigenvalues contains periodic singularities. Using (34), the Floquet exponents can be calculated directly from the solution of the transformed Riccati differential equation. We obtain the following result: $\beta_1 = 0$ and $\beta_2 = -2.07 \cdot 10^6$.

V. Conclusion

The dynamic behavior of a DTL oscillator has been investigated rigorously. To this aim the linear time-varying approximation is used. This method enables a general description of the local dynamic behavior of nonlinear circuits. It has been shown that, in order to find the dynamic eigenvalues of second-order nonlinear circuits, a Riccati differential equation must be solved. Since the solution of the Riccati equation may contain singularities, numerical calculations could be difficult. A variable transformation has been introduced to transform the Riccati equation into a second-order linear time-varying differential equation, which can be solved simultaneously with the calculation of the dynamic bias trajectory. Its use has been shown when applied to the DTL oscillator.

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Quadratic Stabilization of Uncertain Discrete-Time Fuzzy Dynamic Systems

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***Abstract*—New approaches to quadratic stabilization of uncertain discrete-time fuzzy dynamic systems are developed in this paper. This uncertain fuzzy dynamic model is used to represent a class of uncertain discrete-time complex nonlinear systems which include both linguistic information and system uncertainties. It is shown that the uncertain fuzzy dynamic system is stabilizable if a suitable Riccati equation or a set of Riccati equations have solutions. Constructive algorithms are also developed to obtain the stabilization feedback control laws. Finally, an example is given to illustrate the application of the proposed method.**

***Index Terms*—Discrete-time systems, fuzzy control, fuzzy uncertain systems, quadratic stabilization.**

I. Introduction

Fuzzy logic control (FLC) has become more and more popular in industries in recent years. FLC techniques represent a means of both collecting human knowledge and expertise and dealing with uncertainties in the process of control [1]–[5]. Fuzzy control usually decomposes a complex system into several subsystems according to the human expert's understanding of the system and uses a simple control law to emulate the human control strategy in each local operating region. The global control law is then constructed by combining all the local control actions through fuzzy membership functions. Because of the complexities of nonlinear systems, to find a set of local stabilization control actions is much easier than to find a global stabilization control action for the system.

Recently, there have appeared a number of stability-analysis results in fuzzy control literature [6]–[8], where the Takagi–Sugeno fuzzy models [9], [10] are used. The stability of the overall fuzzy system is determined by checking a Lyapunov equation. It is required that a common positive definite matrix P can be found to satisfy the Lyapunov equation for all the local models. However, this is a difficult problem to solve for all but low-ordersystems. Linear-matrix-inequalities (LMI) techniques have been used to avoid such a problem [11]–[13].

During the last few years, we have proposed a number of new methods for the systematic analysis and design of fuzzy logic controllers based on the so-called fuzzy dynamic model or T–S model, which consists of a family of local linear models smoothly connected through fuzzy membership functions [14]–[21]. The basic idea of these methods is to design a feedback controller for each local model and to construct a global controller from the local controllers in such a way that global stability of the closed-loop-fuzzy-control system is guaranteed.

However, it can be observed that in the Takagi–Sugeno models or the dynamic–fuzzy models, each local subsystem is deterministic in the sense that there is no uncertainty considered in the local model. The resulting global model is also deterministic in the same sense.

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