DESIGN OF CURRENT-MODE COMPANDING $\sqrt{}$-DOMAIN DYNAMIC CIRCUITS

J. Mulder W.A. Serdijn A.C. van der Woerd A.H.M. van Roermund

**Abstract** - In this paper, the strong inversion MOS analogue of the log-domain principle, or dynamic translinear principle, is derived. The resulting class of $\sqrt{}$-domain circuits is based on the square law describing the large-signal behaviour of the MOS transistor in strong inversion. An inherent characteristic of $\sqrt{}$-domain filters is instantaneous companding, which is especially interesting for low-voltage applications. The design of a current-controlled two-integrator oscillator, based on the $\sqrt{}$-domain principle, is described. The oscillator was realised in a 1.6 $\mu$m n-well CMOS process, for a supply voltage of 3.3 V. Measurements show a total harmonic distortion of -42 dB and linear frequency tunability across 1.3 decades.

## I Introduction

The recent number of publications on log-domain [1], companding current-mode [2], exponential state-space [3] or dynamic translinear [4] filters, has increased the interest in circuits based on the large-signal characteristics of both the bipolar and the MOS transistor. With respect to MOS technology, the dynamic translinear principle can be applied only in the subthreshold region, where the MOST behaves exponentially. However, in this region, operation is limited to low frequencies only. In strong inversion, the large-signal behaviour is approximately described by the square law. Several basic principles to implement filters based on the square law can be distinguished.

The first group consists of linear transconductances based on the square law. This idea was first introduced in [5]. A number of different elaborations of the basic idea can be distinguished within this group. Filters based on linear MOS transconductances are not companding; the capacitance voltage swings are linearly related to the input signals.

The second group consists of circuits based on the MOS strong inversion analogue of the log-domain principle: the $\sqrt{}$-domain principle [6]. Just like circuits based on the log-domain principle, $\sqrt{}$-domain circuits are inherently instantaneous companding. This property makes them very interesting for low-voltage applications. In current-mode companding circuits, the relative voltage swings across the capacitors are smaller than the input current swings, thus cancelling the linear relation between the maximum input current of the circuit and the supply voltage.

A current-mode approach is best suited to describe instantaneous companding filters based on the large-signal behaviour of the bipolar and the MOS transistor. In Sec. II, both the log-domain principle and its square law analogue, the $\sqrt{}$-domain principle, are explained from a current-mode point of view.

In Sec. III, a companding current-controlled oscillator is designed, based on the $\sqrt{}$-domain principle. The oscillator was realised in a full-custom 1.6 $\mu$m n-well CMOS process. Measurement results are presented in Sec. IV.

## II Companding

Figure 1 shows two generic subcircuits of log-domain and $\sqrt{}$-domain circuits, respectively. The log-domain and $\sqrt{}$-domain principle can be explained quite elegantly with the help of these circuits.

### Exponential law

Translinear circuits are based on the exponential law describing the bipolar transistor or the MOS transistor in weak inversion, given by:

$$I_C = I_s e^{V_{BE}/U_T} \tag{1}$$

J. Mulder, W.A. Serdijn, A.C. van der Woerd and A.H.M. van Roermund are with the Electronics Research Laboratory, Department of Electrical Engineering, Delft University of Technology, Delft, The Netherlands

where all symbols have their usual meaning.

The key to dynamic translinear circuits, or log-domain circuits, are the capacitance *currents.* The capacitance current shown in Fig. 1a can be derived from the time derivative of eqn (1). This yields:

$$I_{cap} = CU_T\frac{\dot{I}_C}{I_C} \tag{2}$$

where $C$ is the capacitance value and the dot represents the time derivative.

This equation shows a nonlinear relation between the collector current $I_C$ and the capacitance current $I_{cap}$. The linear derivative term $CU_T\dot{I}_C$ can be isolated by multiplying eqn (2) by the (strictly positive) denominator $I_C$. The dynamic translinear principle thus states that *the time derivative of a current is equivalent to a product of currents.* This product of currents can be implemented by means of the translinear principle [7]. Because of the key role of the translinear principle, we prefer the term 'dynamic translinear'.

### Square law

In analogy with the dynamic translinear principle, the $\sqrt{}$-domain principle is based on the large-signal behaviour of the MOS transistor operating in strong inversion, given by:

$$I_D = \frac{\beta}{2}(V_{GS} - V_{th})^2 \tag{3}$$

where all symbols have their usual meaning.

Again, the key issue is the current $I_{cap}$ flowing through the capacitance $C$ shown in Fig. 1b. The large-signal equation describing $I_{cap}$ can be obtained from the time derivative of eqn (3). In terms of the drain current $I_D$, $I_{cap}$ is given by:

$$I_{cap} = \frac{C}{\sqrt{2\beta I_D}}\dot{I}_D \tag{4}$$

A better insight into the $\sqrt{}$-domain principle is obtained by slightly rewriting this equation, multiplying it by $\sqrt{I_D}$. This yields:

$$\frac{C}{\sqrt{2\beta}}\dot{I}_D = \sqrt{I_D}I_{cap} \tag{5}$$

The $\sqrt{}$-domain principle thus reads: *The derivative of a current is equivalent to the product of the square root of that current and a capacitance current.* Consequently, a differential equation can be implemented by substituting a product of a current and the square root of a current for each of the derivatives in that differential equation. An algebraic equation without any derivatives is obtained. Implementing this algebraic equation thus becomes equivalent to implementing the differential equation.

In dynamic translinear circuits, the translinear principle is used to implement the required products of currents. In case of the $\sqrt{}$-domain principle, the voltage-translinear principle [8] (a term coined in [7]) can be applied to implement the algebraic equation on the right-hand side of eqn (5). The voltage-translinear principle reads that a loop of gate-source voltages, as shown in Fig. 2, can be described in terms of currents by:

$$\sqrt{I_1} + \sqrt{I_3} = \sqrt{I_2} + \sqrt{I_4} \tag{6}$$

## III Design of a CCO

The log-domain principle and the $\sqrt{}$-domain principle can be used to implement differential equations. To demonstrate the design procedure using the $\sqrt{}$-domain principle, in this section, a current-controlled two-integrator oscillator is synthesised. The oscillator is designed for a supply voltage of 3.3 V.

## $\sqrt{}$-Domain integrator

The most important block to be designed is the integrator. Any integrator can be described by the dimensionless differential equation: $\dot{z} = x$, where the dot represents differentiation with respect to dimensionless time $\tau$, and $x$ and $z$ represent the dimensionless input and output signal, respectively. This dimensionless differential equation has to be transformed into an equation with the proper dimensions for the $\sqrt{}$-domain principle to be applicable. First, information is carried by currents. Therefore, $x$ and $z$ are transformed into the currents $I_{in}$ and $I_{out}$, respectively.

Second, the dimensionless time $\tau$ has to be transformed to the time $t$ with dimension [$s$]. In this transformation, the term $C/\sqrt{2\beta}$ has to be introduced, since this term is on the left-hand side of eqn (5) while the right-hand side of eqn (5) is to be implemented. A possible time transformation is: $t = C\sqrt{I_{o1}}\tau/(\sqrt{2\beta}I_{o2})$, where $I_{o1}$ and $I_{o2}$ are DC bias currents. After these transformations, the differential equation describing the integrator reads:

$$\frac{C\sqrt{I_{o1}}}{\sqrt{2\beta}}\dot{I}_{out} = I_{o2}I_{in} \tag{7}$$

From this equation, the derivative $\dot{I}_{out}$ can be eliminated by applying the $\sqrt{}$-domain principle. The derivative $\dot{I}_{out}$ is thus replaced by an algebraic expression in the variables $I_{out}$ and $I_{cap}$. Equation (5) and Fig. 1b can be used to this end, by substitution of $I_{out}$ for $I_D$. Substituting eqn (5) in (7), an algebraic equation is obtained:

$$\sqrt{I_{out}I_{o1}}I_{cap} = I_{o2}I_{in} \tag{8}$$

In strong inversion MOS, this algebraic equation can be implemented using the voltage-translinear principle. Unfortunately, no general synthesis method exists [8]. The only possible solution is to divide eqn (8) into several simpler functions for which voltage-translinear circuits are known.

A square-root circuit can be used to implement the term $\sqrt{I_{out}I_{o1}}$ [8]. A modified version of the circuit published in [8] is shown in Fig. 3. Due to the up-down topology used for the translinear loop, this circuit is less sensitive to body effect.

Using a square-root circuit, the square root term in eqn (8) is substituted for by the output current $I_z$ of the square-root circuit. The resulting equation can be implemented by the four-quadrant multiplier/divider shown in Fig. 4 [8]. Both the input and the output of this multiplier are differential. The output of the multiplier is the capacitance current of eqn (8): $I_{cap} = I_{o2}I_{in}/I_z$. The multiplier is loaded by a PMOS current mirror to obtain a single-ended output current, which is supplied to the capacitor of the integrator. In the output subcircuit, a PMOS transistor was used, instead of the NMOS transistor shown in Fig. 1b, for reasons of voltage compatibility with the output of the PMOS current mirror.

The overall block schematic of the $\sqrt{}$-domain integrator is shown in Fig. 5.

## Two-integrator oscillator

A two-integrator oscillator can be realised by applying negative feedback to a cascade of two integrators. Figure 6 shows the block schematic of the realised current-controlled companding oscillator. An amplitude control mechanism has to be added to the loop of two integrators and one inverter, in order to obtain a unique limit cycle. By applying negative local feedback around each of the integrators, the oscillation can be damped. Similarly, positive feedback undamps the oscillation. Using multipliers as feedback elements facilitates the application of both positive and negative feedback to the integrators. The input signals of the multiplier are the output signal of the integrator and a control signal, which determines the amount of feedback. This control signal is the difference between the measured amplitude and a reference amplitude. The amplitude of the oscillation can be measured using Pythagoras' law [9], which can be implemented by two voltage-translinear square circuits [8]. The amplitude of the oscillator can be tuned by means of the reference amplitude current.

The oscillation frequency is equal to the unity gain frequency of the designed $\sqrt{}$-domain integrator, which can be found from eqn (7). This shows that the oscillation frequency can be tuned linearly by means of $I_{o2}$.

## IV Measurement results

The oscillator was realised in a 1.6 $\mu$m n-well CMOS process. All DC bias currents and the two capacitors of the oscillator were connected externally. The occupied chip area is 0.65 $mm^2$. The largest part of the chip area is consumed by the NMOS current mirrors of the square root circuit and the multiplier shown in Figs 3 and 4, which were biased in the moderate inversion region, by means of a large aspect ratio, in order to gain some voltage room.

The output currents of the oscillator were measured across two resistors of 100 k$\Omega$, using two external CB current buffers. The DC current used to bias the integrators in class A was 5 $\mu$A. The amplitude of the oscillation was 3.6 $\mu$A, which is 72% of the bias current. The capacitors had a value of 82 pF and the control currents $I_{o1}$ and $I_{o2}$ were 5 $\mu$A and 3.1 $\mu$A, respectively. The measured frequency of 22 kHz compares reasonably well with the calculated frequency of 28 kHz.

The measured frequency spectrum is depicted in Fig. 7. The harmonic distortion was mainly caused by the second and third harmonic at -46 dB and -45 dB, respectively. The total harmonic distortion thus amounts to -42 dB.

The oscillation frequency was linearly tunable by means of the control current $I_{o2}$ across 1.3 decades of current, from about 2.6 kHz to 53 kHz, as is shown in Fig. 8. Linear tuning is prohibited for large values of $I_{o2}$ by the limited supply voltage.

## V Conclusions

Based on the square law, approximately describing the MOS transistor in the strong inversion region, the $\sqrt{}$-domain principle is derived, which is the analogue of the log-domain principle, or dynamic translinear principle.

The $\sqrt{}$-domain principle can be used to substitute algebraic expressions for the derivatives in a differential equation. By eliminating the derivatives, an algebraic current-mode equation results. This algebraic equation can be implemented by means of the voltage-translinear principle.

The $\sqrt{}$-domain principle was used to design a companding current-controlled oscillator, which was realised in a 1.6 $\mu$m n-well CMOS process. The measured distortion of the oscillator was -42 dB. The oscillation frequency was tunable across 1.3 decades.

## References


[1] R.W. Adams. Filtering in the log domain. 63rd Convention A.E.S., LA, preprint 1470, May 1979.

[2] E. Seevinck. Companding current-mode integrator: A new circuit principle for continuous-time monolithic filters. *Elec. Letters*, 26(24):2046–2047, November 1990.

[3] D.R. Frey. Exponential state space filters: a generic current mode design strategy. *IEEE Trans. on CAS I*, 43(1):34–42, January 1996.

[4] J. Mulder, A.C. v.d. Woerd, W.A. Serdijn, and A.H.M. v. Roermund. An RMS-DC converter based on the dynamical translinear principle. In *Proc. European Solid-St. Circ. Conf.*, pages 312–315, 1996.

[5] A. Nedungadi and T.R. Viswanathan. Design of linear CMOS transconductance elements. *IEEE Trans. on CAS*, 31(10):891–894, October 1984.

[6] J. Mulder, A.C. van der Woerd, W.A. Serdijn, and A.H.M. van Roermund. Current-mode companding $\sqrt{x}$-domain integrator. *Elec. Letters*, 32(3):198–199, February 1996.

[7] B. Gilbert. Translinear circuits: an historical overview. *Analog Int. Circ. and Signal Processing*, 9(2):95–118, March 1996.

[8] R.J. Wiegerink. *Analysis and synthesis of MOS translinear circuits.* PhD thesis, Twente University of Technology, Enschede, 1992.

[9] F. Doorenbosch and Y. Goinga. Integrable, wideband, automatic volume control (A.V.C.) using Pythagors's law for amplitude detection. *Elec. Letters*, 12(16):418–420, August 1976.

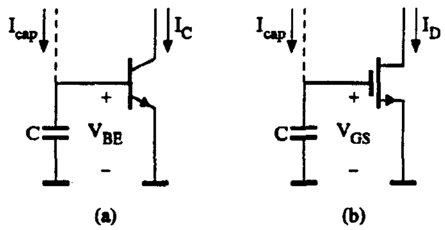


Figure 1: Principle of a) log-domain and b) $\sqrt{}$-domain circuits.

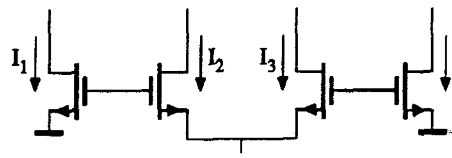

Figure 2: A voltage-translinear loop.

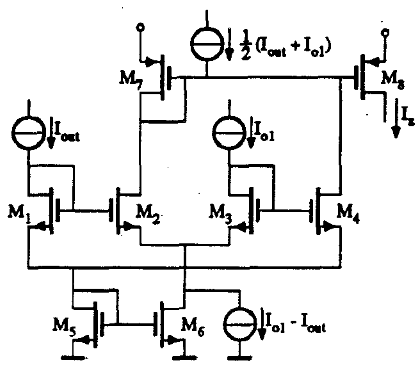

Figure 3: Square root circuit.

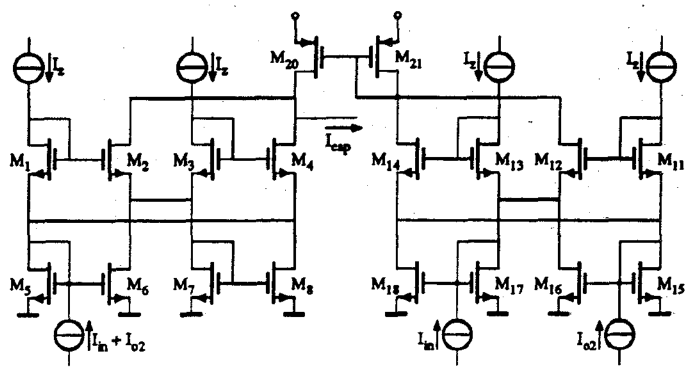

Figure 4: Four-quadrant multiplier/divider.

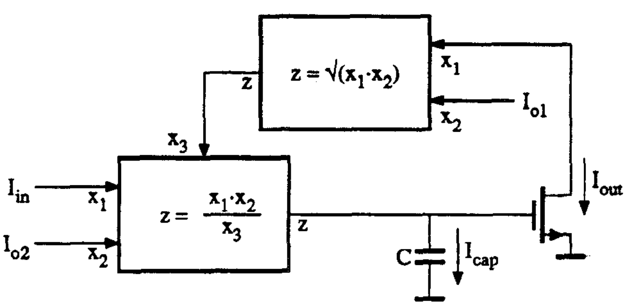


Figure 5: $\sqrt{}$-Domain integrator.

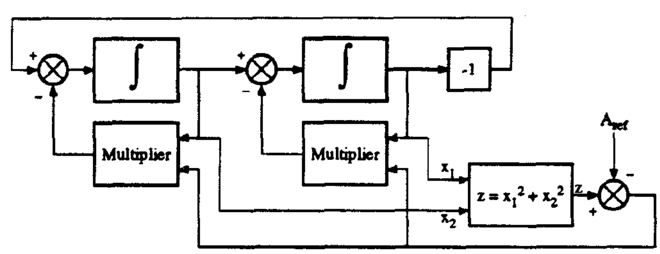


Figure 6: Two-integrator oscillator.

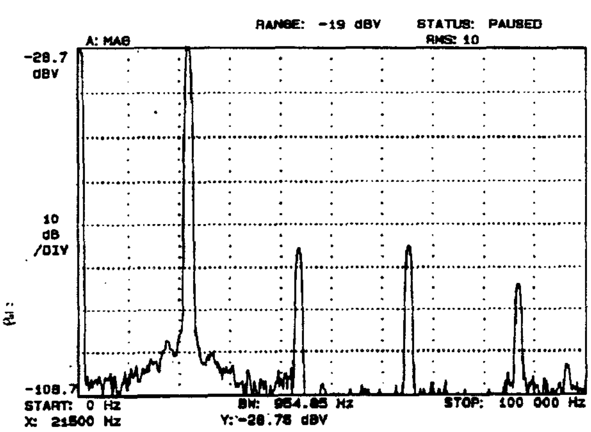


Figure 7: Frequency spectrum of the oscillator.

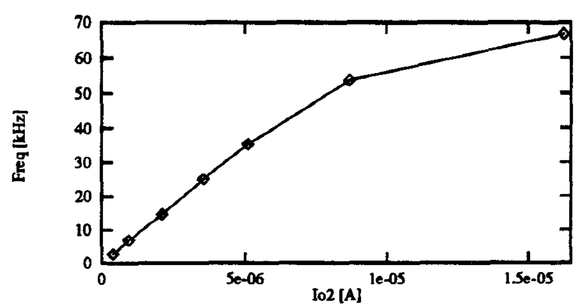


Figure 8: Frequency control of the oscillator.