Mapping Polynomial Differential Equations onto Silicon: the Dynamic Translinear Approach

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Abstract—*A promising new approach to shorten the design trajectory of analog integrated circuits without giving up functionality is formed by the class of dynamic translinear circuits. This paper presents a structured design method for this young, yet rapidly developing, circuit paradigm. As a design example, the design of a low-voltage translinear second-order quadrature oscillator is presented. The circuit is a direct implementation of a nonlinear second-order state-space description, comprises only two capacitors and a handful of bipolar transistors and can be instantaneously controlled over a very wide frequency range by only one control current.*

I. Introduction

Electronics design can be considered to be the mapping of a set of mathematical functions onto silicon. For discrete-time signal-processing systems, of which the digital signal processors (DSPs) today are by far the most popular, this comes down to the implementation of a number of difference equations, whereas for continuous-time signal-processing systems, often denoted by the term analog, differential equations are the starting points. In mixed analog-digital systems, the analog parts, however, often occupy less than ten percent of the complete, i.e., the mixed analog-digital circuitry, whereas their design trajectory is often substantially longer and therefore more expensive than of their digital counterparts. Where does this discrepancy arise from? This can be partially explained by the fact that, at circuit level, for analog circuits far more components play an important role; various types of transistors, diodes, resistors and capacitors, to mention a few; sometimes also inductors, resonators, and others. Whereas for digital circuits, the complete functionality is covered by transistors only[1].

From the above, it automatically follows that, if we restrict ourselves to the use of as few different types of components as possible, without giving up functionality, we can shorten the analog design trajectory considerably, in the same way as this is done for digital circuits. One successful approach, as we will see in this paper, is given by the class of circuits called *dynamic translinear circuits.*

Dynamic translinear (DTL) circuits, of which recently an all-encompassing current-mode analysis and synthesis theory has been developed in Delft [1], are based on the DTL principle, which can be regarded as a generalization of the well-known 'static' translinear principle, formulated by Gilbert in 1975 [2]. The DTL principle can be applied to the structured design of both linear differential equations, i.e. £lters, and non-linear differential equations, e.g., RMS-DC converters, oscillators, phaselock loops (PLLs) and even chaos. In fact, the DTL principle facilitates a direct mapping of any function, described by polynomial differential equations, onto silicon.

Application areas where DTL circuits can be successfully used include audio signal processing, radio-frequency transceivers, infra-red and £ber-optic front-ends and biomedical applications.

This paper aims to present a structured design method for DTL circuits. The static and dynamic TL principles are reviewed in Section 2. Section 3 presents the design method, applied to the design of a DTL quadrature oscillator, starting from a nonlinear second-order state-space description that describes the oscillator behavior in the time domain. After four hierarchical design steps, being dimension transformation, the introduction of capacitance currents, TL decomposition and circuit implementation, a complete circuit diagram results. Simulation results of the thus obtained DTL quadrature oscillator, are presented in Section 4.

II. Design principles

TL circuits can be divided into two major groups: static and dynamic TL circuits. The £rst group can be applied to realize a wide variety of linear and non-linear static transfer functions. All kinds of frequency-dependent functions can be implemented by circuits of the second group. The underlying principles of static and dynamic TL circuits are reviewed in this section.

A. Static translinear principle

TL circuits are based on the exponential relation between voltage and current, characteristic for the bipolar transistor and the MOS transistor in the weak inversion region. In the following discussion, bipolar transistors are assumed. The collector current I_C of a bipolar transistor in the active region is given by:

$$I_C = I_S \mathrm{e}^{\mathrm{V_{BE}}/\mathrm{V_T}}, \tag{1}$$

where all symbols have their usual meaning.

The TL principle applies to loops of semiconductor junctions. A TL loop is characterized by an even number of junctions [2]. The number of devices with a clockwise orientation equals the number of counter-clockwise oriented devices. An example of a four-transistor TL loop is shown in Fig. 1. It is assumed that the transistors are somehow biased at the collector currents I_1 through I_4. When all devices are equivalent and operate at the same temperature, this yields the familiar representation of TL loops in terms of products of currents:

$$I_1 I_3 = I_2 I_4. \tag{2}$$

This generic TL equation is the basis for a wide variety of static electronic functions, which are theoretically temperature and process independent.

[1] It must be noted that, for higher frequencies or bit rates, also the interconnects come into play. However, their in¤uence is considered to be equally important for analog as well as digital systems.

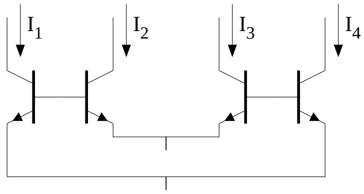

Fig. 1. A four-transistor translinear loop.

B. Dynamic translinear principle

The static TL principle is limited to frequency-independent transfer functions. By admitting capacitors in the TL loops, the TL principle can be generalized to include frequency-dependent transfer functions. The term 'Dynamic Translinear' was coined in [3] to describe the resulting class of circuits. In contrast to other names proposed in literature, such as 'log-domain' [4], 'companding current-mode' [5], 'exponential state-space' [6], this term emphasizes the TL nature of these circuits, which is a distinct advantage with respect to structured analysis and synthesis.

The DTL principle can be explained with reference to the sub-circuit shown in Fig. 2. Using a current-mode approach, this circuit is described in terms of the collector current I_C and the current I_{cap} ¤owing through the capacitance C. Note that the dc voltage source V_{const} does not affect I_{cap}. An expression for I_{cap} can be derived from the time derivative of (1) [3, 5]:

$$I_{\text{cap}} = CV_T \frac{\dot{I}_C}{I_C}, \tag{3}$$

where the dot represents differentiation with respect to time.

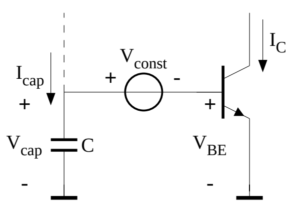

Fig. 2. Principle of dynamic translinear circuits.

Equation (3) shows that I_{cap} is a non-linear function of I_C and its time derivative $\dot{I}_C$. More insight in (3) is obtained by slightly rewriting it:

$$CV_T \dot{I}_C = I_{\text{cap}} I_C. \tag{4}$$

This equation directly states the DTL principle: *A time derivative of a current can be mapped onto a product of currents*. At this point, the conventional TL principle comes into play, since the product of currents on the right-hand side (RHS) of (4) can be realized very elegantly by means of this principle. Thus, the implementation of (part of) a differential equation (DE) becomes equivalent to the implementation of a product of currents.

The DTL principle can be used to implement a wide variety of DEs, describing signal processing functions. For example, £lters are described by linear DEs. Examples of non-linear DEs are harmonic and chaotic oscillators, PLLs and RMS-DC converters.

III. Structured Design of a Translinear Second-Order Quadrature Oscillator

Synthesis of a dynamic circuit, be it linear or non-linear, starts with a polynomial DE or with a set of polynomial DEs describing its function. Often, it is more convenient to use a state-space description, which is mathematically equivalent. The structured synthesis method for DTL circuits is illustrated here by the design of a particular second-order quadrature oscillator, described in the time domain by:

$$\frac{dx_1(\tau)}{d\tau} = g[x_1(\tau)] + x_2(\tau), \tag{5}$$

$$\frac{dx_2(\tau)}{d\tau} = g[x_2(\tau)] - x_1(\tau), \tag{6}$$

where τ is the dimensionless time of the oscillator and g is a (nonlinear) odd-symmetry function of the two quadrature oscillator signals x_1 and x_2.

A. Transformations

In the pure mathematical domain, equations are dimensionless. However, as soon as we enter the electronics domain to £nd an implementation of the equation, we are bound to quantities having dimensions. In the case of TL circuits, all time-varying signals in the DEs and the tunable parameters, have to be transformed into currents. Hence, the signals x_1 and x_2 can be transformed into the currents I_1 and I_2 through the equations: $x_1 = I_1/I_O$ and $x_2 = I_2/I_O$, I_O being a bias current that determines the absolute current swings.

The dimensionless time τ can be transformed into the time t with dimension $[s]$ through the equation [1]:

$$\frac{d}{d\tau} = \frac{CV_T}{I_O}\frac{d}{dt}. \tag{7}$$

From (7), two important characteristics of DTL oscillators can be derived. First, time (t) is inversely proportional to current I_O. This means that the oscillator will be linearly frequency tunable by means of only one control current. Second, this control current must be proportional to the absolute temperature to eliminate the in¤uence of the temperature on the oscillator.

Applying the above transformations, the resulting current-mode multi-variable polynomial DEs become:

$$CV_T \dot{I}_1 = I_O^2 h(I_1, I_O) + I_O I_2, \tag{8}$$

$$CV_T \dot{I}_2 = I_O^2 h(I_2, I_O) - I_O I_1, \tag{9}$$

where $h(I_i, I_O)$ equals $g(I_i/I_O)$.

Since the currents in this state-space description will equal the currents in the £nal oscillator circuit, at this point it is already possible to determine the most important oscillator characteristics, which are its oscillation frequency ω_{osc} and its amplitude $\hat{I}_{\text{osc}} = \hat{I}_1 = \hat{I}_2$. If we assume that the oscillator currents are sinusoidal, thus $I_1 = \hat{I}_{\text{osc}} \sin(\omega_{\text{osc}} t + \theta_{\text{osc}})$ and $I_2 = \hat{I}_{\text{osc}} \cos(\omega_{\text{osc}} t + \theta_{\text{osc}})$, ω_{osc} and $\hat{I}_{\text{osc}}$ follow from:

$$\omega_{\text{osc}} = \frac{I_O}{V_T C}, \tag{10}$$

$$\int_0^{T/2} h(I_i, I_O)\, dt = 0, \tag{11}$$

where T equals $2\pi/\omega_{\text{osc}}$.

B. De£nition of the capacitance current

Conventional TL circuits are described by multivariable polynomials, in which all variables are currents. The gap between these current-mode polynomials and the DEs can be bridged by the introduction of capacitance currents, since the DTL principle states that a derivative can be replaced by a product of currents.

De£ning I_{cap1} and I_{cap2} as $I_{\text{cap1}} = CV_T \frac{\dot{I}_1}{I_1+I_O}$ and $I_{\text{cap2}} = CV_T \frac{\dot{I}_2}{I_2+I_O}$, the above state-space description transforms into:

$$(I_1 + I_O)I_{\text{cap1}} = I_O^2 h(I_1, I_O) + I_O I_2, \quad (12)$$

$$(I_2 + I_O)I_{\text{cap2}} = I_O^2 h(I_2, I_O) - I_O I_1. \quad (13)$$

From this point on, the synthesis theory for static TL circuits can be used [7], since both sides of the above DEs are now described by current-mode multivariable polynomials.

C. Translinear decomposition

The above set of polynomials is the basis of the next synthesis step, which is TL decomposition. That is, the polynomials have to be mapped onto a set of TL loop equations that are each characterized by the general equation: $\prod_{\text{CW}} J_{C,i} = \prod_{\text{CCW}} J_{C,i}$, $J_{C,i}$ being the transistor collector current densities in clockwise (CW) or counter-clockwise (CCW) direction.

One possible solution is achieved by 'parametric' decomposition of (12) and (13). Two intermediate currents, I_{y_1} and I_{y_2} are introduced, resulting in:

$$I_O(I_O + I_{y_1}) = (I_1 + I_O)(I_{\text{cap1}} + I_O), \quad (14)$$

$$I_O(I_O + I_{y_2}) = (I_2 + I_O)(I_{\text{cap2}} + I_O), \quad (15)$$

$$I_{y_1} = \quad I_2 + I_O k(I_1, I_O), \quad (16)$$

$$I_{y_2} = -I_1 + I_O k(I_2, I_O), \quad (17)$$

with $k(I_i, I_O) = \frac{I_i}{I_O} + h(I_i, I_O)$.

From its de£nition, it follows that $k(I_i, I_O)$ must be a nonlinear time-invariant odd-symmetry function of I_i and I_O, whose derivative $k'(I_i, I_O)$ with respect to I_i is larger than one for small values of $|I_i|$ and smaller than one for large values of $|I_i|$. Possible polynomial functions are the ones characterized by: $(1 + x)^m(1 - y)^n = (1 - x)^m(1 + y)^n$, $m, n \in \mathcal{N}$, $m > n$, which solves to

$$y = k(x) = \frac{\left(\frac{1+x}{1-x}\right)^{m/n} - 1}{\left(\frac{1+x}{1-x}\right)^{m/n} + 1}. \quad (18)$$

These functions are easily implemented in TL circuits [8], the simplest one implementing (18) for $m = 2$ and $n = 1$ by a third-order TL loop. See Fig. 3.

Using

$$k(I_i, I_O) = G\frac{\left(\frac{I_O+I_i}{I_O-I_i}\right)^2 - 1}{\left(\frac{I_O+I_i}{I_O-I_i}\right)^2 + 1} = 2G\frac{I_O I_i}{I_O^2 + I_i^2}, \quad (19)$$

$G > \frac{1}{2}$ being a constant, we arrive at the following £nal TL

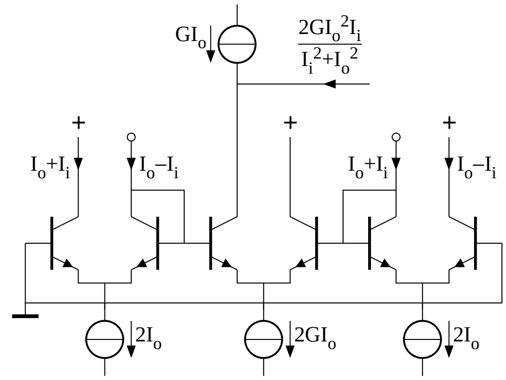

Fig. 3. Implementation of the TL function $k(I_i, I_O)$.

decomposition:

$$I_O(I_O + I_{y_1}) = (I_1 + I_O)(I_{\text{cap1}} + I_O), \quad (20)$$

$$I_O(I_O + I_{y_2}) = (I_2 + I_O)(I_{\text{cap2}} + I_O), \quad (21)$$

$$(I_O + I_1)^2(I_G + I_2 - I_{y_1}) = (I_O - I_1)^2(I_G - I_2 + I_{y_1}), \quad (22)$$

$$(I_O + I_2)^2(I_G + I_1 + I_{y_2}) = (I_O - I_2)^2(I_G - I_1 - I_{y_2}), \quad (23)$$

with $I_G = GI_O$.

Assuming the oscillator output currents to be sinusoidal, the oscillator amplitude $\hat{I}_{\text{osc}}$ follows from (11), (16) and (19), which yields

$$\hat{I}_{\text{osc}} \approx I_O \frac{\sqrt{6}}{2}\sqrt{2G - 1}. \quad (24)$$

Note that $\hat{I}_{\text{osc}}$ is indeed proportional to I_O as has been discussed previously.

D. Circuit implementation

The last synthesis step is the circuit implementation. The TL decomposition that was found during the previous synthesis step has to be mapped onto a TL circuit topology and the correct collector currents have to be forced through the transistors. Biasing methods for bipolar all-NPN TL topologies are presented in [7].

A possible biasing arrangement for the TL quadrature oscillator, assuming ideal current sources, is depicted in Fig. 4. Transistors Q_{13}–Q_{16}, Q_1–Q_4, Q_{16}–Q_{21} and Q_4–Q_9 implement (20)–(23), respectively. Current sources GI_O and $2GI_O$ are current controlled. Q_{24} and Q_{10} deliver the oscillator output currents I_1 and I_2.

Replacing all the ideal sources by practical ones yields the £nal circuit diagram, which, for the sake of brevity, is not discussed here.

IV. Simulation results

The £nal circuit was simulated using SPICE and realistic (IC) capacitor and (minimum-size) transistor models of our in-house 1-μ, 15-GHz, bipolar IC process. Typical transistor parameters are: $h_{\text{fe,NPN}} \approx 100$, $f_{T,\text{NPN}} \approx 15$ GHz, $h_{\text{fe,LPNP}} \approx 55$ and $f_{T,\text{LPNP}} \approx 80$ MHz. The results indicate the correct operation of the TL quadrature oscillator for various temperatures and values of I_O, G ($> \frac{1}{2}$) and C_1 (= C_2), yielding oscillations from 50 mHz ($C_1 = C_2 =$

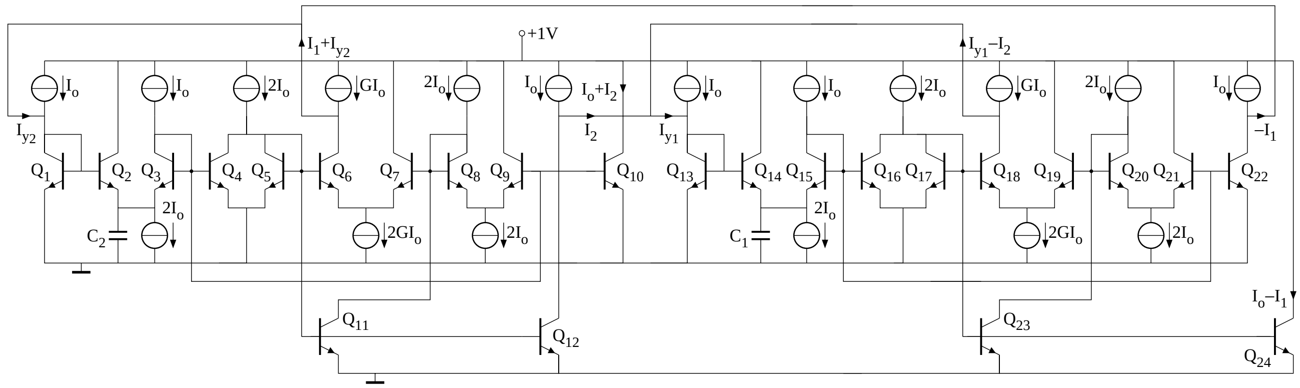

Fig. 4. Possible biasing arrangement for the 1-volt TL quadrature oscillator. Ideal current sources are assumed.

1 nF, I_O = 10 pA) up to 13 MHz (C_1 = C_2 = 0.1 pF, I_O = 10 μA), in accordance with (10) and (24). The supply voltage can be as low as 0.95 V. The current consumption approximately equals $25 + 4G$ times I_O. For G = 0.6, the total harmonic distortion is below 2 %. For I_O = 1 μA, C = 100 pF and G = 0.7, the quadrature phase error equals 0.31 degrees.

Fig. 5 depicts the oscillation frequency f_{osc} (in Hz) as a function of control current I_O for £ve different integratable capacitor values: 0.1 pF, 1 pF, 10 pF, 100 pF and 1 nF. G equals 0.7. From this plot, it can be deduced that this particular TL quadrature oscillator can be controlled over a very wide frequency range of 6 (!) decades.

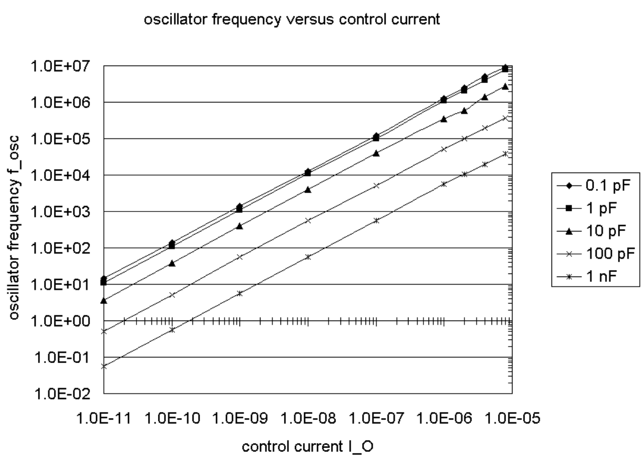

Fig. 5. Simulated oscillation frequency as a function of control current I_O for £ve different capacitor values.

Since k is a time-invariant function of I_O and I_i, $i \in [1, 2]$, the output current waveform is independent of the oscillation frequency. This has been veri£ed by means of a Fourier analysis and proved to be true for the complete 'linear' current range, i.e., between 10 pA and 2 μA. Fig. 6 depicts the output frequency spectrum of the oscillator running at 50 kHz. The total harmonic distortion is mainly determined by the second and third harmonic and equals 1.4 % or -37 dB. A smaller G will lower the distortion even further.

V. Conclusions

In this paper, it was shown that dynamic translinear circuits constitute an exciting new approach to the structured design of analog signal processing functions, using transistors and capacitors only. The presented design

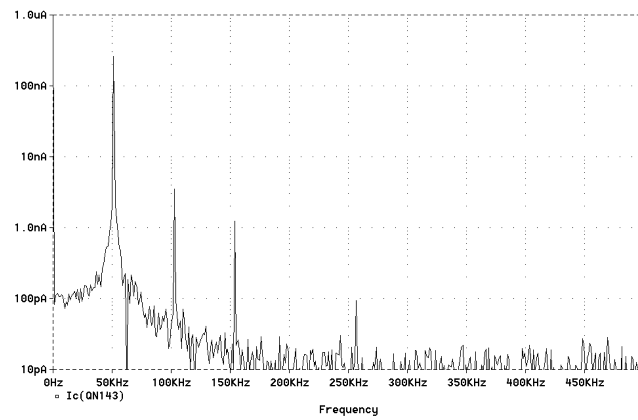

Fig. 6. Simulated output spectrum for f_{osc} = 50 kHz.

methodology was elaborated into the design of a translinear second-order quadrature oscillator starting from a nonlinear second-order state-space description. The resulting circuit comprises only two capacitors and handful of transistors and can be instantaneously controlled over a very wide frequency range by only one control current (I_O). Its harmonic distortion is directly related to another parameter (G) and can be made small by the design.

Simulations indicate that the proposed circuit operates from a single supply voltage down to 1 V, oscillates over 8.4 decades of frequency with less than 2 % total harmonic distortion. The quadrature phase error equals 0.31 degrees.

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