THE LINEAR TIME-VARYING APPROACH APPLIED TO A FIRST-ORDER DYNAMIC TRANSLINEAR FILTER

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ABSTRACT

Dynamic translinear (DTL) circuits use the exponential input-output relation of the transistor as a primitive for the synthesis of electronic circuits. As a consequence the analysis of the dynamic behaviour of this type of circuits in the presence of parasitics results in analyzing dynamic nonlinear circuits.

There are three different approaches that can be applied: the linear time-invariant (LTI) approach, the quasi-static (QS) approach and the linear time-varying (LTV) approach. The LTI approach is limited to circuits in which the bias points are fixed. The QS approach can be applied to circuits with time-varying bias points, but these time-variations must be relatively slow. The LTV approach has none of these restrictions and opens the way to a general and structured design method for dynamic nonlinear circuits.

We apply the linear time-varying approach to a first-order dynamic translinear filter. Its dynamic eigenvalue is calculated and stability is determined by means of the corresponding Floquet exponent. We also apply the quasi-static approach to the same DTL filter. Comparisons between the linear time-varying approach and the quasi-static approach show the limitations of the quasi-static approach and the usefulness of the linear time-varying approach in the design of DTL circuits.

1. INTRODUCTION

In modern electronic circuits the classic linear design methodologies are limiting the possibilities for large dynamic range, low power consumption, low voltage operation and large bandwidth. The obvious way to go beyond these limitations is the use of a nonlinear design approach, using nonlinear primitives. The dynamic translinear (DTL) approach is such a design methodology [1, 2, 3]. In DTL circuits the exponential input-output relation of the bipolar transistor (or the MOS transistor in weak inversion) is used as a primitive for synthesis. Since DTL circuits operate in the current domain, they are relatively insensitive to collector-substrate and wiring capacitors. So a relatively good HF behaviour can be expected. However much about the precise HF behaviour is still unknown.

DTL synthesis and analysis are based on the nonlinear input-output relation of the transistor. Therefore the system behaviour in the presence of parasitics is often defined by nonlinear differential equations, even if the ideal overall transfer function is linear. These equations do not give much insight. In order to handle these nonlinear dynamic effects in a structured design trajectory we need a systematic methodology to model the dynamic behaviour of nonlinear circuits. There are three different approaches that can be applied: the linear time-invariant (LTI) approach, the quasi-static (QS) approach and the linear time-varying (LTV) approach.

In the *LTI approach* the nonlinear relationships are approximated by a first-order Taylor expansion around a fixed bias point. This results in a LTI small-signal model, which enables the use of Laplace transforms and a description of the internal dynamics in terms of the poles and zeros of the transfer function. If the variation of the signals is small compared to the bias point the resulting description is accurate.

The *QS approach* is a straightforward extension of the LTI approach in which the bias point is assumed to be signal-dependent. However it is assumed that the variations of the bias point are slow compared to the dynamics of the system (slowly varying signals). As a consequence the dynamics of the system can be studied in every point of the signal-dependent bias trajectory independently ("frozen time approach"). This results in a description in terms of time-varying quasi-static poles and zeros. This model is only valid for slowly varying signals [7].

The *LTV approach* is a general and accurate method to describe the dynamic behaviour of nonlinear circuits. Again the nonlinear relationships are approximated by a first-order Taylor expansion around a signal-dependent bias trajectory, but now the time-dependency of the bias point is not frozen when studying the dynamic behaviour. The resulting LTV small-signal model is a consistent generalization of the LTI small-signal model, in which the eigenvalue and pole concept are generalized by means of the dynamic eigenvalue and Floquet exponent [4, 5, 6].

In this paper we apply the linear time-varying approach to a first-order DTL filter. In Section 2 we give a short overview of the linear time-varying approach. The linear time-invariant approach, quasi-static approach and linear time-varying approach are applied to a first-order DTL filter in Section 3 and some comparisons are made. Finally some conclusions are given in Section 4.

2. THE LINEAR TIME-VARYING APPROACH

The linear time-varying approach models the behaviour of a nonlinear circuit in the vicinity of a time-varying bias trajectory. When determining the time-varying bias trajectory no distinction is made between bias and signal components, which implies that also class-B circuits and oscillators can be treated. In this section we give a short overview of the LTV approach for nonlinear circuits with first-order dynamics, since this is all the theory needed for the example worked out in this paper. A more general description of the LTV approach can be found in [4, 5, 6].

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We begin with a state-space description of the circuit:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f[x(t), u(t)]. \qquad (1)$$

Here $x(t)$ represents the state variable and $u(t)$ the external source.

The state-space description (1) can be used to calculate the time-varying bias trajectory $x_b(t)$ of the state variable as function of the external source $u_b(t)$. This signal-dependent bias trajectory x_b is called the *dynamic bias trajectory*.

The dynamic behaviour of the circuit in the vicinity of the dynamic bias trajectory can be modeled by studying small variations $x_\delta(t)$ around the dynamic bias trajectory $x_b(t)$. The noise behaviour can be modeled by studying small variations $u_\delta(t)$ on the deterministic external signal $u_b(t)$. In studying the dynamic behaviour it suffices to deal with $u_\delta(t) = 0$. By linearizing the state-space description (1) around $x_b(t)$ and $u_b(t)$ we obtain the following *homogeneous variational equation* for $x_\delta(t)$

$$\frac{\mathrm{d}x_\delta(t)}{\mathrm{d}t} = a_x[x_b(t), u_b(t)] \cdot x_\delta(t) \triangleq a_x(t) \cdot x_\delta(t). \qquad (2)$$

in which $a_x(t)$ is the Jacobian of f with respect to the state-variable x.

The dynamic behaviour of the nonlinear circuit for a given input is determined by the dynamic eigenvalue $\lambda(t)$ of the homogeneous variational equation. The solution $x_\delta(t)$ is of the form:

$$x_\delta(t) = s \cdot e^{\gamma(t)} = s \cdot e^{\int_0^t \lambda(\tau)\mathrm{d}\tau} \qquad (3)$$

where the one-dimensional eigenvector s can be chosen to equal unity. The *dynamic eigenvalue* $\lambda(t)$ is simply given by:

$$\lambda(t) = a_x(t) \qquad (4)$$

If the deterministic input signal $u_b(t)$ of the nonlinear circuit is chosen to be periodic then $a_x(t)$ is periodic too. In this case we can apply the Floquet theory of periodic LTV systems to the variational equation. If we apply the definition of the Floquet exponent β with trajectory of operation $x_b(t)$, $u_b(t)$ of [8] then

$$\beta = \frac{1}{T}\int_0^t \lambda(\tau)\mathrm{d}\tau. \qquad (5)$$

The system is stable if the real part of the Floquet exponents β is negative.

For LTI small-signal circuits the eigenvalues are constant and the Floquet exponents work out to the eigenvalues, which for LTI systems are equivalent to the system poles. Thus the Floquet exponents are a generalization of the traditional pole concept, as far as stability is concerned.

3. ANALYSIS OF A FIRST-ORDER DTL FILTER

In this section we introduce a linear first-order DTL filter. The circuit topology is extended in order to eliminate the influence of most of the parasitic capacitors. Only one dominant parasitic capacitor remains. We apply the LTI, QS and LTV approach to that circuit.

3.1. Circuit description

A possible implementation of a linear first-order filter using the dynamic translinear principle is depicted in Figure 1. More details on the synthesis of this circuit can be found in [2]. It has the following cut-off frequency ω_c:

$$\omega_c = \frac{I_0}{C \cdot V_T} \qquad (6)$$

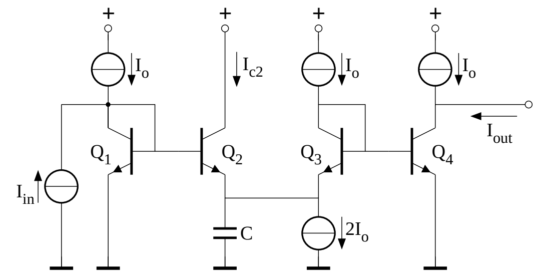

Figure 1: Implementation of a linear first-order DTL filter

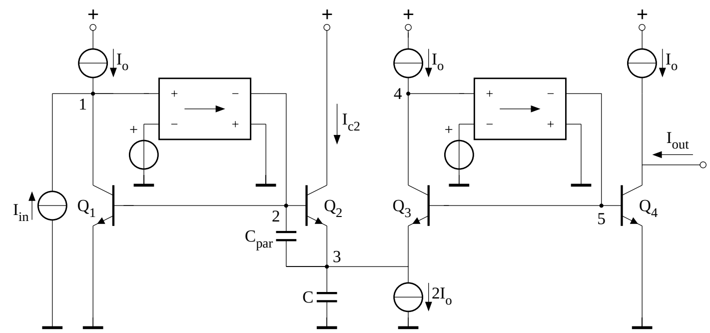

Figure 2: Modified linear first-order DTL filter

which is tunable by the bias current I_0.

The circuit behaviour for high frequencies is influenced by parasitic capacitors (collector-substrate, base-emitter and base-collector capacitors). Measures are taken to make their influence negligible. The resulting circuit is shown in Figure 2. Only the influence of C_{par} (the base-emitter capacitance of transistor Q2) can not be counteracted by proper circuit design. Therefore its influence on the circuit HF behaviour is further analyzed.

3.2. The linear time-invariant and quasi-static approach

We first analyze the DTL filter in Figure 2 by using a linear time-invariant small-signal model. Then we apply the quasi-static approach, by assuming that the bias point and therefore the parameters of the small-signal model are signal dependent.

3.2.1. The LTI approach

In the LTI approach we assume the signals to be small compared to the DC bias currents. The circuit is linearized in its DC bias point by replacing the transistors with simplified hybrid-π models. After applying the nullor conditions we obtain the LTI small-signal model of Figure 3. In this figure I_{in} and I_{out} are the input and output current (which are assumed to be small), i_{c1} and i_{c2} are the small-signal collector currents of transistors Q1 and Q2, gm_1, gm_2 and gm_4 are the transconductance of transistors Q1, Q2 and Q4, C is the external capacitor and C_{par} is the base-emitter capacitance of transistor $Q2$.

The input-output relation in the Laplace domain is given by:

$$\frac{I_{out}(s)}{I_{in}(s)} = \frac{gm_4}{gm_1} \cdot \frac{1 + s\frac{C_{par}}{gm_2}}{1 + s\frac{C+C_{par}}{gm_2}} \qquad (7)$$

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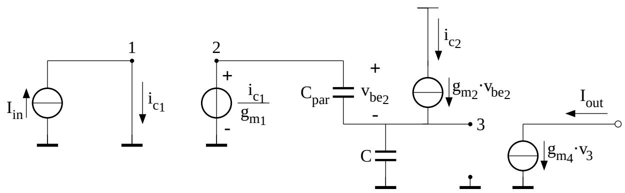

Figure 3: LTI small-signal model of the modified DTL filter

The pole of the system equals:

$$p = -\frac{gm_2}{C + C_{par}} = -\frac{I_0}{V_T(C + C_{par})} \triangleq -\frac{I_0}{\alpha + \beta} \qquad (8)$$

where $\alpha = V_T\, C$ and $\beta = V_T\, C_{par}$. Note that the cut-off frequency is reduced due to the parasitic capacitor (compare with (6)) and that a zero appears at $z = -I_0/\beta$. The LTI approach gives us a first feeling for the circuit behaviour, but can not be applied for large input signals.

3.2.2. *The QS approach*

We drop the assumption that the signals should be small compared with the bias currents, but suppose the signals to be slowly varying. Then we can apply the QS approach by substituting the bias point of the LTI small-signal model by a signal dependent bias trajectory. We obtain

$$\frac{I_{out}(s)}{I_{in}(s)} = \frac{gm_4[I_{out}(t) + I_0]}{gm_1[I_{in}(t) + I_0]} \cdot \frac{1 + s\frac{C_{par}}{gm_2[I_{c2}(t)]}}{1 + s\frac{C + C_{par}}{gm_2[I_{c2}(t)]}} \qquad (9)$$

Since the parameter C_{par} is a junction capacitor it is virtually current independent. The time-varying quasi-static pole of the system equals:

$$p(t) = -\frac{I_{c2}(t)}{\alpha + \beta} \qquad (10)$$

Note that this pole varies with time if the large-signal current $I_{c2}(t)$ varies with time. This is the case even if $\beta = 0$, which corresponds to a parasitic capacitor C_{par} equal to zero.

3.3. The linear time-varying approach

If the slowly varying condition is dropped then the QS approach is not applicable. In that case only the LTV approach is a consistent generalization of the LTI small-signal model. We will now apply the LTV approach to the first-order DTL filter under consideration.

3.3.1. *The state-space description*

We use the Ebers-Moll large-signal model of the bipolar transistor to find the nonlinear differential equation describing the circuit behaviour. By applying Kirchhoffs current law at node 3 of Figure 2 and using the translinear loop equation we obtain:

$$-I_{c2} - \beta\,\frac{\dot I_{c2}}{I_{c2}} + \alpha\,\frac{\dot I_{out}}{I_{out} + I_0} + 2I_0 - I_0 = 0 \qquad (11)$$

$$I_{c2} \cdot (I_{out} + I_0) = (I_{in} + I_0) \cdot I_0 \qquad (12)$$

By substituting (12) into (11) and choosing I_{out} as the state-variable we arrive at the following state-space description:

$$\dot x = f(x,u) \qquad x = I_{out} \qquad u = I_{in} \qquad (13)$$

$$f(x,u) = \frac{1}{\alpha+\beta}\left[\left(\frac{\beta\,\dot u}{u + I_0} - I_0\right)\cdot x + u\,I_0 + \frac{\beta\,\dot u\,I_0}{u + I_0}\right]$$

which is a linear time-varying differential equation.

3.3.2. *The dynamic bias trajectory*

In order to find the dynamic bias trajectory we need to specify the input signal. We choose the following sinusoidal input signal:

$$u_b = I_0\,\xi\,\sin(\omega_s t) \qquad (14)$$

where I_0 is the bias current, ξ is the modulation depth ($\xi \in [0,1]$) and ω_s is the radial frequency.

The dynamic bias trajectory $x_b(t)$ equals the solution of the state-space description (13) for the chosen input signal $u_b = I_0\xi\sin(\omega_s t)$. This is a first-order linear time-varying differential equation and in this special case we can find the following analytical expression for the dynamic bias trajectory:

$$x_b(t) = [1 + \xi\sin(\omega_s t)]^{\frac{\beta}{\alpha+\beta}}\, e^{-\frac{I_0}{\alpha+\beta}t}\, I_0 \times \qquad (15)$$

$$\times\left\{ I_0 + \frac{I_0}{\alpha+\beta}\int_0^t \left[I_0\xi\sin(\omega_s\tau) + \frac{\beta\omega_s\xi\cos(\omega_s\tau)}{1+\xi\sin(\omega_s\tau)}\right] \times \right.$$

$$\left. \times\,[1 + \xi\sin(\omega_s\tau)]^{-\frac{\beta}{\alpha+\beta}}\, e^{\frac{I_0}{\alpha+\beta}\tau}\,\mathrm{d}\tau \right\}$$

where we have chosen the initial condition $x_b(0) = I_0$ since for an input current equal to zero at $t = 0$ the output current equals $I_{out}(0) = x_b(0) = I_0$.

3.3.3. *The linear time-varying small-signal model*

The homogeneous variational equation is obtained by inserting (13) and (14) in (1) and (2). This yields:

$$\dot x_\delta = a_x(t)\cdot x_\delta = \frac{1}{\alpha+\beta}\left[\beta\,\frac{\omega_s\xi\cos(\omega_s t)}{1+\xi\sin(\omega_s t)} - I_0\right]\cdot x_\delta \qquad (16)$$

Using (4) the dynamic eigenvalue $\lambda(t)$ is given by:

$$\lambda(t) = \frac{1}{\alpha+\beta}\left[\beta\,\frac{\omega_s\,\xi\cos(\omega_s t)}{1+\xi\sin(\omega_s t)} - I_0\right] \qquad (17)$$

To show the significance of the dynamic eigenvalue we consider two special cases. If $\beta \to 0$, that is, if the parasitic capacitance C_{par} becomes negligible, then the dynamic eigenvalue approaches the constant pole $p = -I_0/(C\,V_T)$ of the ideal overall transfer function (compare with (6)). This occurs even if the time-variations of the input-signal are on the same order of magnitude as the ideal time-constant of the system.

If $\omega_s \to \infty$ then the amplitude of the dynamic eigenvalue goes to infinity. Physically this is explained by a signal bypass, caused by the parasitic capacitor C_{par}. This corresponds to a constant transfer. Note that in the LTI small-signal model of the circuit this behaviour occurs in the frequency range above the pole and zero.

3.3.4. *The Floquet exponent*

The Floquet exponent is obtained by substituting (17) into (5):

$$\beta = \frac{1}{T}\int_0^T \lambda(\tau)\,\mathrm{d}\tau = -\frac{I_0}{\alpha+\beta} \qquad (18)$$

Thus for any sinusoidal input source the dynamic bias trajectory is stable, since for any input frequency or amplitude β is negative. Note that the Floquet exponent equals the pole of the LTI small-signal model (see (8)).

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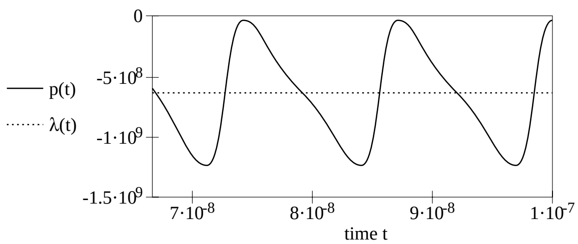

Figure 4: Dynamic eigenvalue λ(t) (dotted) and quasi-static pole p(t) as function of time t (f_s = 80MHz, C_{par} =0.01pF)

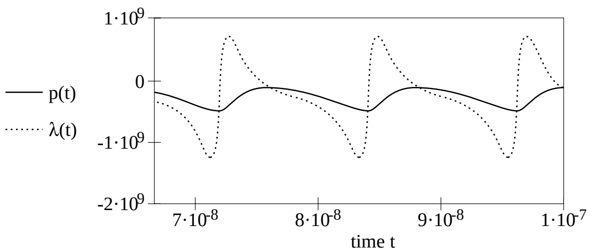

Figure 5: Dynamic eigenvalue λ(t) (dotted) and quasi-static pole p(t) as function of time t (f_s = 80MHz, C_{par} = 5pF)

3.4. Comparison between the quasi-static and linear time-varying approach

The example worked out shows that the dynamic eigenvalue converges to the ideal linear pole if the parasitic capacitor vanishes. This convergence takes place independently of the time-variations of the input signal in comparison to the internal dynamics of the system. Thus the dynamic eigenvalue reflects the fact that the overall transfer becomes linear, which is a desirable property when designing the dynamic behaviour. The quasi-static pole does not have this property and is only a good model if we deal with slowly-varying signals.

We illustrate the difference between the two approaches through some numerical examples of the linear first-order DTL-filter. Suppose that an ideal cut-off frequency of 100MHz is specified. We choose the external capacitor to be C = 5pF, which implies that I_0 = 81.7μA (see (6)).

First we apply a sinusoid to the input with a frequency of f_s = 80MHz. We choose a modulation depth ξ of 0.97, which ensures that the circuit operates in its nonlinear region. Thus the input amplitude equals 79.25μA. The dynamic eigenvalue and the quasi-static pole are plotted for a parasitic capacitor C_{par} of 0.01pF in Figure 4 and of 5pF in Figure 5. Figure 4 shows that for a very small parasitic capacitor the dynamic eigenvalue is almost time-invariant and converges to the ideal linear pole. The quasi-static pole does not converge. Figure 5 shows that for C_{par} = 5pF the dynamic eigenvalue even becomes positive. The Floquet exponent however is negative, thus the system is stable.

Then in Figure 6 the frequency of the input signal is chosen to be f_s = 800kHz, the input amplitude remains 79.25μA and C_{par} = 5pF. Then we deal with a slowly-varying system and we see that the quasi-static pole is equal to the dynamic eigenvalue.

4. CONCLUSIONS AND FUTURE WORK

Dynamic translinear circuits use the exponential input-output relation of the transistor a primitive for the synthesis of electronic

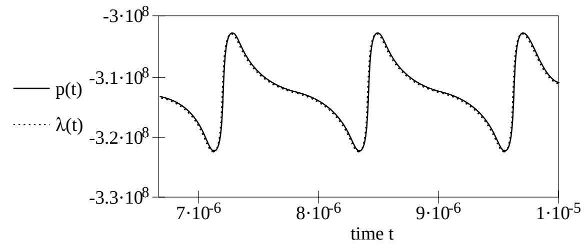

Figure 6: Dynamic eigenvalue λ(t) (dotted) and quasi-static pole p(t) as function of time t (f_s = 800kHz, C_{par} = 5pF)

circuits. As a consequence the analysis and synthesis of DTL circuits in the presence of parasitics necessitates the modeling of the dynamic behaviour of nonlinear circuits.

Through the example of a first-order linear DTL filter it has been shown that the linear time-varying approach is a general and suitable method. In this model the eigenvalue and pole concept of linear time-invariant small-signal circuits are generalized using dynamic eigenvalues and Floquet exponents. The dynamic eigenvalue of the DTL filter was shown to converge to the designed ideal linear pole if the parasitics vanish, which is a desirable property. The time-varying pole of the quasi-static approach does not have this property. If the DTL filter is operating under slowly-varying conditions the quasi-static pole was shown to be equal to the dynamic eigenvalue.

A next step is the investigation of the linear time-varying approach for higher-order DTL circuits and DTL circuits with a designed nonlinear transfer. This topic is presently under research.

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