A Low-Voltage Translinear Second-Order Quadrature Oscillator

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Abstract— **This paper describes the design of a low-voltage translinear second-order quadrature oscillator. The circuit is a direct implementation of a nonlinear second-order state-space description and follows from a recently developed synthesis method for dynamic translinear circuits. It comprises only two capacitors and a handful of bipolar transistors and can be instantaneously controlled over a very wide frequency range by only one control current, which indicates its suitability for spread-spectrum communications. Its total harmonic distortion can be made small by the design. Simulations, using realistic transistor models of a 1-μ, 15-GHz, bipolar IC process, indicate that the oscillator operates from a single supply voltage, which can be as low as 1 V and oscillates over 8.4 decades of frequency (from 50 mHz to 13 MHz) with less than 2 % total harmonic distortion. Its quadrature phase error equals 0.31 degrees.**

I. Introduction

Recently, both an analysis method and a synthesis method for dynamic translinear (DTL) circuits were proposed by the authors [1, 2]. The DTL principle can be regarded as a generalization of the well-known 'static' translinear (TL) principle, formulated by Gilbert in 1975 [3].

An important subclass of DTL circuits is the class of 'translinear filters,' also called 'log-domain' or 'exponential state-space' filters, investigated by various researchers. See [4] for a comprehensive list of DTL publications. However, the DTL principle is not limited to filters, i.e. linear differential equations (DEs). The DTL principle can be applied to the structured design of both linear DEs, i.e. filters, and non-linear DEs, e.g., RMS-DC converters [6], oscillators [7, 8], phaselock loops (PLLs) [9] and even chaos. In fact, the DTL principle facilitates a direct mapping of **any** function, described by polynomial DEs, onto silicon.

Today, many radio communication systems (e.g. GSM and DECT) use complex digital modulation methods. One of the main challenges in integrated complex (de)modulators is the generation of an accurate 90^0 phase difference between the two quadrature signals required by the modulation scheme. Examples of commonly used quadrature generators are

- the combination of a single oscillator, running at the doubled frequency, and two oppositely triggered flip-flops,
- the combination of a single oscillator and a passive phase-shift (e.g., RC-CR,) network [10],
- a four-stage ring oscillator [11],
- a quadrature coupled system of two first-order oscillators [12],
- a quadrature coupled system of two resonator oscillators [10], and
- a two-integrator oscillator.

The latter three quadrature generators have in common that they implement a second-order DE, which ideally equals $\ddot{x}(t) + x(t) = 0$, $x(t)$ being one of the quadrature oscillator signals. The dot represents differentiation with respect to the time t.

The quadrature relation becomes visible only when we split this equation into two equivalent first-order DEs, to obtain the following state-space description:

$$\dot{x}_1(t) = \quad x_2(t), \tag{1}$$
$$\dot{x}_2(t) = -x_1(t), \tag{2}$$

$x_1(t)$ and $x_2(t)$ being the two quadrature oscillator signals.

In order to compensate for the effects of non-idealities in the oscillator circuitry which cause the oscillator amplitudes to be unstable, such as noise and drift, all practical second-order oscillators somehow implement the (nonlinear) second-order DE $\ddot{x}(t) + f[x(t)]\dot{x}(t) + x(t) = 0$, where $f(x)$ is an arbitrary (nonlinear) even-symmetry function of x. When $f(x) > 0$, the oscillator is damped and the amplitude decreases. When $f(x) < 0$, the oscillator is undamped and the amplitude increases.

A general state-space description of a practical second-order quadrature oscillator is thus given by:

$$\dot{x}_1(t) = \quad g_1[x_1(t), x_2(t)], \tag{3}$$
$$\dot{x}_2(t) = -g_2[x_2(t), x_1(t)], \tag{4}$$

where g_1 and g_2 are (nonlinear) odd-symmetry functions of x_1 and x_2. For example, in the case that the amplitude stabilization is implemented by a pythagorator [13], g_1 and g_2 ensure that $x_1^2(t) + x_2^2(t)$ remains constant.

An alternative quadrature oscillator arises if the equivalent functions g_i have only one input variable x_i and are applied in two local feedback loops, according to:

$$\dot{x}_1(t) = g[x_1(t)] + \; x_2(t), \tag{5}$$
$$\dot{x}_2(t) = g[x_2(t)] - x_1(t), \tag{6}$$

where $g = g_1 = g_2$. See Fig. 1.

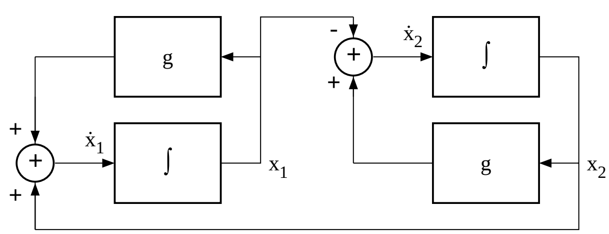

Fig. 1. Principle of a quadrature oscillator based on two odd-symmetry functions arranged in two local feedback loops.

In this paper, we present the design and simulations of a 1-V wide-tunable TL second-order quadrature oscillator, based on the latter principle. The circuit, which comprises only two capacitors and a handful of transistors, is a direct implementation of a nonlinear second-order state-space description by means of the synthesis method proposed in [2] and is tuned by only one control current.

The organization of the paper is as follows. Section 2 introduces the DTL principle, which subsequently is elaborated into the design of the second-order quadrature oscillator in Section

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3, following a systematic approach. The simulation results of the TL quadrature oscillator are discussed in Section 4. Finally, Section 5 deals with the conclusions.

II. Dynamic translinear principle

The key to the DTL principle, from a current-mode point of view, are the capacitance currents. We therefore concentrate on the simple class-A substructure, depicted in Fig. 2. Assuming a bipolar transistor, it follows [1]:

$$CV_T\dot{I}_C = I_C I_{\text{cap}}, \tag{7}$$

C, V_T, I_C and I_{cap} being the capacitance value, the thermal voltage kT/q, the collector current and the capacitance current, respectively. The dot again represents differentiation with respect to time. Note that the DC voltage source V_{const} does not influence the capacitance current.

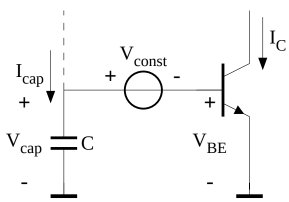

Fig. 2. Principle of dynamic translinear circuits.

From this equation, it can be seen that a time derivative in a DE can be replaced by a product of two currents. This product of currents can be elegantly realized by means of the TL principle [3].

III. Quadrature oscillator design

The design of the second-order DTL quadrature oscillator starts with the dimensionless state-space description, similar to (5) and (6), describing the oscillator behavior in the time domain:

$$\frac{dx_1(\tau)}{d\tau} = g[x_1(\tau)] + x_2(\tau), \tag{8}$$

$$\frac{dx_2(\tau)}{d\tau} = g[x_2(\tau)] - x_1(\tau), \tag{9}$$

where τ is the dimensionless time of the oscillator.

A. Transformations

The first synthesis step is the transformation of the above two first-order DEs such that the dimensions of the resulting equations are suitable for a TL realization. In DTL circuits, all signals are currents. Hence, the signals x_1 and x_2 can be transformed into the currents I_1 and I_2 through the equations: $x_1 = I_1/I_O$ and $x_2 = I_2/I_O$, I_O being a bias current that determines the absolute current swings.

The dimensionless time τ can be transformed into the time t with dimension $[s]$ through the equation [2]:

$$\frac{d}{d\tau} = \frac{CV_T}{I_O}\frac{d}{dt}. \tag{10}$$

From (10), two important characteristics of DTL oscillators can be derived. First, time (t) is inversely proportional to current I_O. This means that the oscillator will be linearly frequency tunable by means of only one control current. Second, this control current must be proportional to the absolute temperature to eliminate the influence of the temperature on the oscillator.

Applying the above transformations, the resulting current-mode multi-variable polynomial DEs become:

$$CV_T\dot{I}_1 = I_O^2 h(I_1, I_O) + I_O I_2, \tag{11}$$

$$CV_T\dot{I}_2 = I_O^2 h(I_2, I_O) - I_O I_1, \tag{12}$$

where $h(I_i, I_O)$ equals $g(I_i/I_O)$.

Since the currents in this state-space description will equal the currents in the final oscillator circuit, at this point it is already possible to determine the most important oscillator characteristics, which are its oscillation frequency ω_{osc} and its amplitude $\hat{I}_{\text{osc}} = \hat{I}_1 = \hat{I}_2$. If we assume that the oscillator currents are sinusoidal, thus $I_1 = \hat{I}_{\text{osc}}\sin(\omega_{\text{osc}}t + \theta_{\text{osc}})$ and $I_2 = \hat{I}_{\text{osc}}\cos(\omega_{\text{osc}}t + \theta_{\text{osc}})$, ω_{osc} and $\hat{I}_{\text{osc}}$ follow from:

$$\omega_{\text{osc}} = \frac{I_O}{V_T C}, \tag{13}$$

$$\int_0^{T/2} h(I_i, I_O)dt = 0, \tag{14}$$

where T equals $2\pi/\omega_{\text{osc}}$.

B. Definition of the capacitance currents

The next synthesis step is the elimination of the derivatives. In the previous section, we saw that a derivative can be replaced by a product of a capacitance current and a collector current according to (7). Defining I_{cap1} and I_{cap2} as $I_{\text{cap1}} = CV_T\frac{\dot{I}_1}{I_1+I_O}$ and $I_{\text{cap2}} = CV_T\frac{\dot{I}_2}{I_2+I_O}$, the above state-space description transforms into:

$$(I_1 + I_O)I_{\text{cap1}} = I_O^2 h(I_1, I_O) + I_O I_2, \tag{15}$$

$$(I_2 + I_O)I_{\text{cap2}} = I_O^2 h(I_2, I_O) - I_O I_1. \tag{16}$$

C. Translinear decomposition

The above set of polynomials is the basis of the next synthesis step, which is TL decomposition. That is, the polynomials have to be mapped onto a set of TL loop equations that are each characterized by the general equation: $\prod_{\text{CW}} J_{C,i} = \prod_{\text{CCW}} J_{C,i}$, $J_{C,i}$ being the transistor collector current densities in clockwise (CW) or counter-clockwise (CCW) direction. To this end, the synthesis methods for static TL circuits expounded in [14] can be used.

One possible solution is achieved by 'parametric' decomposition of (15) and (16). Two intermediate currents, I_{y_1} and I_{y_2} are introduced, resulting in:

$$I_O(I_O + I_{y_1}) = (I_1 + I_O)(I_{\text{cap1}} + I_O), \tag{17}$$

$$I_O(I_O + I_{y_2}) = (I_2 + I_O)(I_{\text{cap2}} + I_O), \tag{18}$$

$$I_{y_1} = \quad I_2 + I_O k(I_1, I_O), \tag{19}$$

$$I_{y_2} = -I_1 + I_O k(I_2, I_O), \tag{20}$$

with $k(I_i, I_O) = \frac{I_i}{I_O} + h(I_i, I_O)$.

From its definition, it follows that $k(I_i, I_O)$ must be a nonlinear time-invariant odd-symmetry function of I_i and I_O, whose derivative $k'(I_i, I_O)$ with respect to I_i is larger than one for small values of $|I_i|$ and smaller than one for large values of $|I_i|$. Possible polynomial functions are the ones characterized by: $(1+x)^m(1-y)^n = (1-x)^m(1+y)^n$, $m,n \in \mathcal{N}$, $m > n$, which solves to

$$y = k(x) = \frac{\left(\frac{1+x}{1-x}\right)^{m/n} - 1}{\left(\frac{1+x}{1-x}\right)^{m/n} + 1}. \tag{21}$$

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These functions are easily implemented in TL circuits [5], the simplest one implementing (21) for $m = 2$ and $n = 1$ by a third-order TL loop. See Fig. 3.

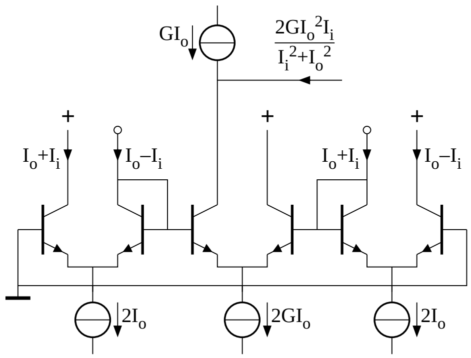

Fig. 3. Implementation of the TL function $k(I_i, I_O)$.

Using

$$k(I_i, I_O) = G\,\frac{\left(\frac{I_O+I_i}{I_O-I_i}\right)^2 - 1}{\left(\frac{I_O+I_i}{I_O-I_i}\right)^2 + 1} = 2G\frac{I_O I_i}{I_O^2 + I_i^2}, \qquad (22)$$

$> G \ \frac{1}{2}$ being a constant, we arrive at the following final TL decomposition:

$$I_O(I_O + I_{y_1}) = (I_1 + I_O)(I_{\mathrm{cap1}} + I_O), \qquad (23)$$

$$I_O(I_O + I_{y_2}) = (I_2 + I_O)(I_{\mathrm{cap2}} + I_O), \qquad (24)$$

$$(I_O + I_1)^2(I_G + I_2 - I_{y_1}) = (I_O - I_1)^2(I_G - I_2 + I_{y_1}), \qquad (25)$$

$$(I_O + I_2)^2(I_G + I_1 + I_{y_2}) = (I_O - I_2)^2(I_G - I_1 - I_{y_2}), \qquad (26)$$

with $I_G = GI_O$.

Assuming the oscillator output currents to be sinusoidal, the oscillator amplitude $\hat{I}_{\mathrm{osc}}$ follows from (14), (19) and (22), which yields

$$\hat{I}_{\mathrm{osc}} \approx I_O \frac{\sqrt{6}}{2}\sqrt{2G - 1}. \qquad (27)$$

Note that $\hat{I}_{\mathrm{osc}}$ is indeed proportional to I_O as has been discussed previously.

D. Circuit implementation

The last synthesis step is the circuit implementation. The TL decomposition that was found during the previous synthesis step has to be mapped onto a TL circuit topology and the correct collector currents have to be forced through the transistors. Biasing methods for bipolar all-NPN TL topologies are presented in [14].

A possible biasing arrangement for the TL quadrature oscillator, assuming ideal current sources, is depicted in Fig. 4. Transistors Q_{13}–Q_{16}, Q_1–Q_4, Q_{16}–Q_{21} and Q_4–Q_9 implement (23)–(26), respectively. Current sources GI_O and $2GI_O$ are current controlled. Q_{24} and Q_{10} deliver the oscillator output currents I_1 and I_2.

Replacing all the ideal sources by practical ones yields the circuit diagram depicted in Fig. 5. Current mirrors with multiple outputs (transistors Q_{25}–Q_{42}, Q_{46}, Q_{50}–Q_{57}) produce replicas of I_O. Q_{43}–Q_{45} and Q_{47}–Q_{49} enlarge the loop gain, thereby reducing the influence of the base currents. The constant factor G is set by the emitter areas of Q_{26}, Q_{32}, Q_{38} and Q_{54}.

IV. Simulation results

The circuit shown in Fig. 5 was simulated using SPICE and realistic (IC) capacitor and (minimum-size) transistor models of our in-house 1-μ, 15-GHz, bipolar IC process. Typical transistor parameters are: $h_{\mathrm{fe,NPN}} \approx 100$, $f_{T,\mathrm{NPN}} \approx 15$ GHz, $h_{\mathrm{fe,LPNP}} \approx 55$ and $f_{T,\mathrm{LPNP}} \approx 80$ MHz. The results indicate the correct operation of the TL quadrature oscillator for various temperatures and values of I_O, G ($> \frac{1}{2}$) and C_1 (= C_2), yielding oscillations from 50 mHz (C_1 = C_2 = 1 nF, I_O = 10 pA) up to 13 MHz (C_1 = C_2 = 0.1 pF, I_O = 10 μA), in accordance with (13) and (27). The supply voltage can be as low as 0.95 V. The current consumption approximately equals 25+4 G times I_O. For G = 0.6, the total harmonic distortion is below 2 %. For I_O = 1 μA, C = 100 pF and G = 0.7, the quadrature phase error equals 0.31 degrees.

Fig. 6 depicts the oscillation frequency f_{osc} (in Hz) as a function of control current I_O for five different integratable capacitor values: 0.1 pF, 1 pF, 10 pF, 100 pF and 1 nF. G equals 0.7. From this plot, it can be deduced that this particular TL quadrature oscillator can be controlled over a very wide frequency range of 6 (!) decades.

Deviations from the theoretically linear relationship between I_O and f_{osc} at the high end of the frequency range are caused by high-level injection in the lateral PNPs. With their SPICE parameter IKF set to default, instead of 10 μA, the maximum oscillation frequency equals 90 MHz.

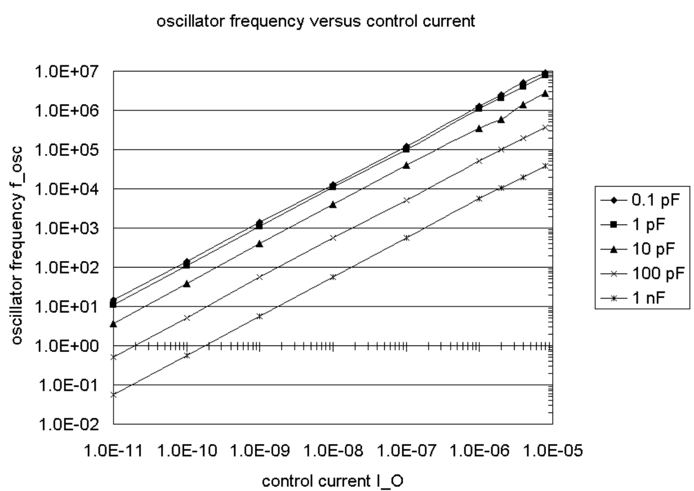

Fig. 6. Simulated oscillation frequency as a function of control current I_O for five different capacitor values.

The favorable property of a very wide frequency range makes the TL oscillator an interesting candidate for frequency synthesizers, such as those needed in, e.g., spread-spectrum receivers and transmitters.

Since k is a time-invariant function of I_O and I_i, $i \in [1, 2]$, the output current waveform is independent of the oscillation frequency. This has been verified by means of a Fourier analysis and proved to be true for the complete 'linear' current range, i.e., between 10 pA and 2 μA. Fig. 7 depicts the output frequency spectrum of the oscillator running at 50 kHz. The total harmonic distortion is mainly determined by the second and third harmonic and equals 1.4 % or -37 dB. A smaller G will lower the distortion even further.

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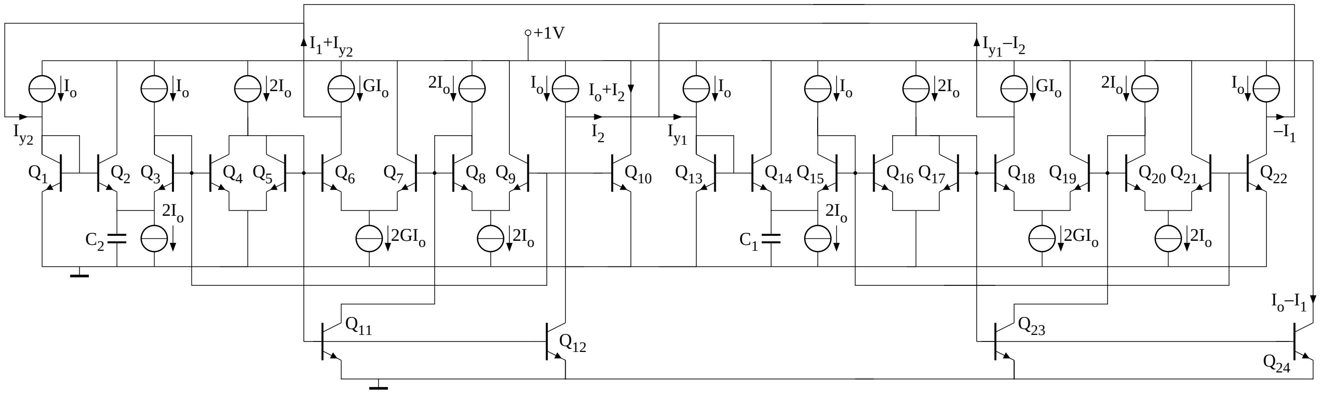

Fig. 4. Possible biasing arrangement for the 1-volt TL quadrature oscillator. Ideal current sources are assumed.

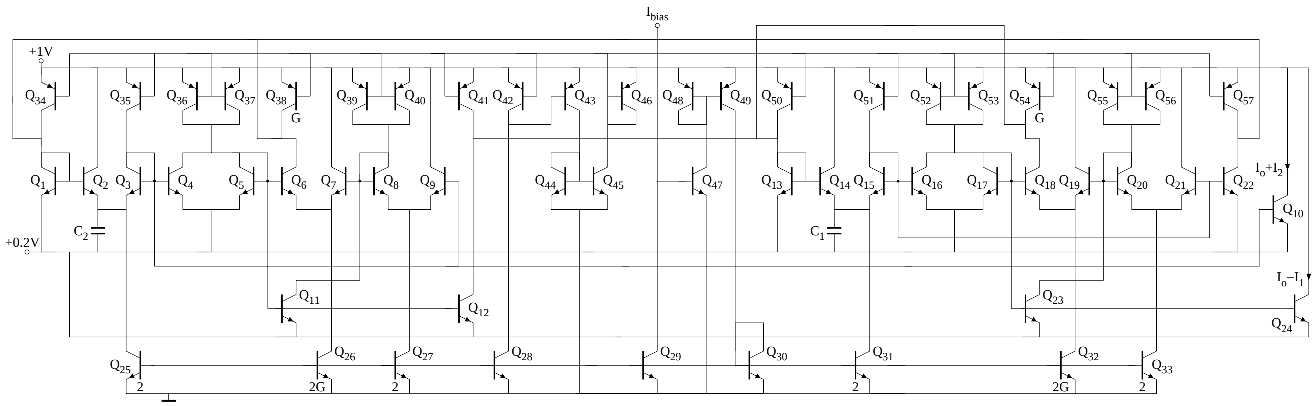

Fig. 5. Complete circuit diagram of the 1-volt TL quadrature oscillator.

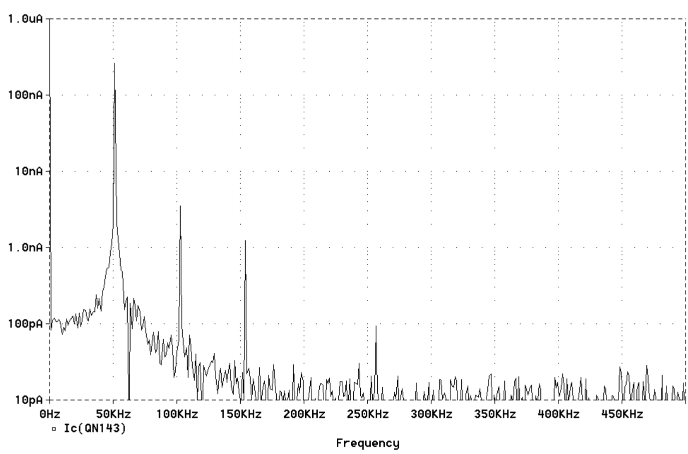

Fig. 7. Simulated output spectrum for $f_{\rm osc}$ = 50 kHz.

V. Conclusions

A translinear second-order quadrature oscillator has been introduced. The circuit is a direct implementation of a nonlinear second-order state-space description. It comprises only two capacitors and handful of transistors and can be instantaneously controlled over a very wide frequency range by only one control current (I_O), which indicates that the translinear oscillator is an interesting candidate for spread-spectrum frequency synthesizers. Its harmonic distortion is directly related to another parameter (G) and can be made small by the design.

Simulations indicate that the proposed circuit operates from a single supply voltage down to 1 V, oscillates over 8.4 decades of frequency with less than 2 % total harmonic distortion. The quadrature phase error equals 0.31 degrees.

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