ANALYSIS OF NOISE IN HIGHER-ORDER TRANSLINEAR FILTERS

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Abstract—**Noise analysis of higher-order translinear filters cannot be established through straight-forward extension of analysis techniques for first-order TL filters, due to the presence noise sources that interact with multiple filter states simultaneously, in a nonlinear and non-stationary way. Such noise sources, that are not encountered in in first-order TL filters, require a new analysis technique. This paper presents a noise analysis technique for higher-order TL filters that that takes this specific 'higher-order' noise into account. An example analysis for a second-order TL low-pass filter is given.**

I. INTRODUCTION

Translinear (TL) filters are currently considered as a promising approach in analog electronics to meet the challenging demands for low-power low-voltage operation and full integratability. Owing to their explicit use of the large-signal, exponential V-I characteristic of the bipolar transistor, such filters allow considerably lower biasing current- and voltage levels than conventional continuous-time filters, and also provide linear tuning of important filter parameters over a wide-range.

The noise behavior of TL filters, characterized by important specifications as the Dynamic Range (DR) and the maximal Signal-to-Noise Ration (SNR), also differs considerably from that of conventional continuous-time filters. Although such filters are externally linear, they are internally nonlinear. Internally generated circuit noise is therefore generally subjected to nonlinear processing, resulting in phenomena as signal×noise intermodulation and non-stationary noise [1, 2].

Approaches for the analysis of noise in TL/companding filters [3–5] have so far mainly concentrated on first-order TL filters. Some treatises have suggested [4, 6] that noise in higher-order TL/companding filters can be analyzed through straight-forward extension of the techniques available for first-order TL filters. However, such an approach cannot be entirely correct, since it overlooks 'higher-order TL filter noise': noise sources that interact with *multiple* capacitors (filter states) in a nonlinear and non-stationary way.

This paper presents a noise analysis technique for higher-order TL filters that, unlike previous techniques, accounts for 'higher-order TL filter noise'.

The approach is based on circuit analysis techniques for dynamic translinear circuits [7], summarized in Section 2. Section 3 outlines the general principles of the proposed analysis technique, while Section 4 focuses on the analysis of 'higher-order TL filter noise'. Section 5 illustrates the procedure for a second order TL low-pass filter.

II. TRANSLINEAR PRINCIPLES

An important step in an analysis of TL filter noise is the determination of the (generally nonlinear) relation between the filter output signal and the various noise contributions inside the circuit. The static translinear (STL) and dynamic (DTL) translinear principles, outlined below, provide a straight-forward technique to obtain this relation.

The static TL principle applies to loops of even numbers of semiconductor junctions [8], like the one depicted in Fig. 1(a). Assuming that the transistors are somehow bi-

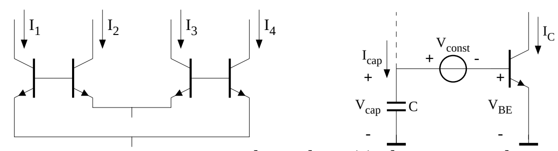

Fig. 1. Four-transistor static translinear loop (a), dynamic translinear principle (b).

ased at currents I_1-I_4, it follows from the exponential V_{be}-I_c relations that

$$I_1 I_3 = I_2 I_4, \quad (1)$$

i.e. *the product of currents through the junctions connected in one direction equals the product of currents through the junctions in the other direction*.

The dynamic TL principle [7] applies to loops of capacitors and semiconductor junctions, as depicted in Fig. 1(b), and therefore provides a generalization of the STL principle to frequency-dependent transfers. According to this figure, the relation between the capacitance current I_{cap} and the collector current I_c can be written as:

$$C U_T \dot{I}_c = I_{\mathrm{cap}} I_c. \quad (2)$$

Thus, *a time derivative of a current can be mapped onto a product of currents*. By way of this product of currents, the DTL principle can be combined with the STL principle to analyze and synthesize TL filters, and other DTL circuits.

Higher-order TL circuits can be efficiently analyzed and synthesized with the aid of state-space descriptions (SSD). The advantage of such descriptions is that they allow an n-th order differential equation (DE) to be expressed as a set of n simple first-order DEs. Often, voltage-mode SSDs are used [9]. However, since TL circuits are most efficiently analyzed and synthesized in the current-domain, a current-mode SSD is preferable. The DTL and STL principles can be used to construct such a current-mode SSD. Instead of the capacitor voltages, the associated output currents of the (exponential-based) transconductors are used as state-variables. In this way, all state-variables can be represented by (combinations of) collector currents. For example, the collector current of the transistor (exponential transconductor) in Fig. 1(b) is a suitable current-mode state-variable, associated to the capacitor voltage.

For TL filters, that are externally linear, the current-mode SSD, with state-variables I_{x_1}-I_{x_n}, is also linear. It can be expressed in terms of the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, having constant

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coefficients, as:

$$U_T \begin{pmatrix} C_1 \dot{I}_{x_1} \\ \vdots \\ C_n \dot{I}_{x_n} \end{pmatrix} = \mathbf{A} \begin{pmatrix} I_{x_1} \\ \vdots \\ I_{x_2} \end{pmatrix} + \mathbf{B} I_{\text{in}}, \qquad (3)$$

$$I_{\text{out}} = \mathbf{C}\vec{I}_x + \mathbf{D} I_{\text{in}}.$$

III. NOISE ANALYSIS PRINCIPLES

The objective of the presented noise analysis technique is the determination of the equivalent output noise and output SNR due to internal circuit noise. The procedure followed to reach that objective, which is based on an approach [2] for first-order TL filters, is outlined in this section.

• Since STL and DTL analyses are elaborated in the current-domain, the first step in the noise analysis of TL circuits is the transformation of the dominant transistor noise contributions to the transistor collector currents. The two important noise contributions in TL circuits are collector shot noise, dominating at low current levels, and thermal base resistance noise, dominating at high current levels [2]. Other noise contributions are usually negligible [2, 5].

Due to the large-signal operation of the transistors, the collector shot noise is generally nonstationary. Since the stationary thermal base voltage noise is much smaller than the thermal voltage U_T, it can, in first-order approximation, be transformed into a collector noise current through a signal-dependent transconductance $g_m(t)$. The resulting transformed noise current is therefore generally nonstationary too.

The resulting circuit representation concentrates all circuit noise in the collector currents, as illustrated by i_1-i_4 in Fig. 2.

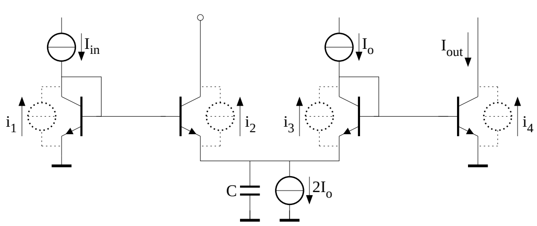

Fig. 2. Representation of noise in a translinear filter.

• In the second step, the TL loop equations and a current-mode SSD in the presence of noise are determined. The TL loop equations relate the (noisy) collector currents to the capacitor currents I_{cap}, the bias and signal current. For example, the STL principle yields the following TL loop equation for the circuit in Fig. 2:

$$(I_{\text{in}} + i_1)(I_o + i_3) = (I_o + I_{\text{cap}} + i_2)(I_{\text{out}} + i_4), \qquad (4)$$

which clearly demonstrates the presence of signal×noise intermodulation in TL filters. A current-mode SSD, in terms of collector currents (including noise) and their time-derivatives, is subsequently obtained from the TL loop equations by application of the DTL principle to the capacitor currents I_{cap}.

• The third step is simplification of the resulting SSD w.r.t. the noise current sources. Although the SSD of TL filters is linear w.r.t. the state-variables, it is generally nonlinear w.r.t. the noise currents, as a result of the internal nonlinear circuit behavior.

Fortunately, since the noise currents are generally small compared to the signal currents, such that noise×noise terms are negligible w.r.t. signal×noise terms, an approximate expression for the equivalent output noise can be obtained by a first-order multi-dimensional Taylor expansion of the SSD to all internal noise sources and their time-derivatives. The resulting SSD is linear time-variant w.r.t. the noise: it 'feeds' the n filter states with the m noise currents through a signal-dependent (and thus time-dependent) $n \times m$ matrix $\mathbf{G}(t)$:

$$U_T \begin{pmatrix} C_1 \dot{I}_{x_1} \\ \vdots \\ C_n \dot{I}_{x_n} \end{pmatrix} = \mathbf{A}\vec{I}_x + \mathbf{B} I_{\text{in}} + \mathbf{G}(t)\vec{i}, \qquad (5)$$

where $\vec{i}$ represents the m noise currents.

• In the fourth step, the average power density spectrum (PSD) of the equivalent output noise is determined through transformation of all noise currents i_1-i_m to the filter output. Two types of noise are distinguished in this respect: noise that interacts with only one filter state, and noise that interacts with multiple states (higher-order TL filter noise).

The first type of noise is similar to the noise encountered in first-order TL filters. The column in $\mathbf{G}(t)$ associated to a noise source $i_k(t)$ of this type contains only one nonzero entry, denoted by $g_{l,k}(t)$. Therefore, the non-stationary noise $g_{l,k}(t)i_k(t)$ can be transformed from the input of state l to the filter output through a single linear time-invariant (LTI) transfer $H_l(\mathrm{j}\omega)$. The contribution to the average output noise PSD, $S_{\text{out},k}(\omega)$, can be obtained by application of the well-known Wiener-Kintchine theorem, since the LTI nature of $H_l(\mathrm{j}\omega)$ allows to interchange the order of the averaging and transformation operations applied to $g_{l,k}i_k$. Thus, when $S_{l,k}(\omega)$ denotes the average PSD of $g_{l,k}(t)i_k(t)$, then:

$$S_{\text{out},k}(\omega) = |H_l(\mathrm{j}\omega)|^2 \, S_{l,k}(\omega). \qquad (6)$$

The second type of noise is not encountered in first-order TL filters. The column in $\mathbf{G}(t)$ associated to a noise source of this type contains multiple, unequal nonzero entries, which means that several correlated non-stationary noise processes propagate to the filter output through different paths. Therefore, as discussed in Section 4, the contribution of this noise to the average output noise PSD cannot be determined with the standard Wiener-Kintchine theorem. Instead, an extended transformation is required.

IV. HIGHER-ORDER TL FILTER NOISE

This section presents an approach to determine the contribution to the average output noise PSD of higher-order TL filter noise, which affects multiple filter states in a nonlinear and non-stationary way. As discussed previously, the columns of $\mathbf{G}(t)$ in eqn (5) associated to this noise contain multiple nonzero entries.

For the moment, consider a noise source $i_k(t)$ that affects the states l and p of a TL filter. Then, the elements $g_{l,k}(t)$

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and $g_{p,k}(t)$ of $\mathbf{G}(t)$ are nonzero. If $i_{\mathrm{out},k}(t)$ denotes the time-domain equivalent output noise due to i_k, and $h_l(\tau)$, $h_p(\tau)$ denote the impulse responses of the transfers from the input of state l and p to the filter output respectively, then:

$$i_{\mathrm{out},k} = h_l * (g_{l,k} i_k) + h_p * (g_{p,k} i_k)\,, \tag{7}$$

where '$*$' represents a convolution. In order to obtain the average PSD of the (non-stationary) noise $i_{\mathrm{out},k}$, the autocorrelation $R_{\mathrm{out},k}(t,\tau)$ between the values of $i_{\mathrm{out},k}$ at times $t_1 = t + \tau$ and $t_2 = t$, defined as:

$$R_{\mathrm{out},k}(t,\tau) \stackrel{\mathrm{def}}{=} \mathrm{E}\left[i_{\mathrm{out},k}(t+\tau) i_{\mathrm{out},k}(t)\right], \tag{8}$$

where $\mathrm{E}[.]$ denotes the expectation operator, has to be determined. This is achieved by substitution of eqn (7) into eqn (8). The resulting expression for $R_{\mathrm{out},k}(t,\tau)$ equals the addition of four correlation functions, each corresponding to another permutation of the indices l and p, of the type:

$$R_{\mathrm{out},l,p,k}(t,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_l(u) h_p(v) \mathrm{E}\big[g_{l,k}(t+\tau-u) \\ g_{p,k}(t-v) R_k\left(t-v, \tau-u+v|I_{\mathrm{in}}\right)\big]_{I_{\mathrm{in}}} \mathrm{d}u\mathrm{d}v, \tag{9}$$

where $R_x(t,\tau|I_{\mathrm{in}})$ denotes the autocorrelation of i_k, conditioned on the filter input signal I_{in}[1]. The expectation in eqn (9), which becomes a time-average in case of a deterministic input signal I_{in}, represents a (cross) correlation function, denoted by $R_{l,p,k}(t,\tau)$, of the non-stationary noise processes $g_{l,k} i_k$ and $g_{p,k} i_k$ supplied to the filter states.

An expression for the average output noise PSD $S_{\mathrm{out},k}(\omega)$ is obtained from eqn (9) by application of a two-dimensional Fourier transform: one transform, with frequency variable ω_1, to the time-difference τ, and the other transform, with frequency variable ω_2, to the absolute time t. Thus, ω_1 is associated to the usual filtering operation, while ω_2 is associated to the non-stationary components in the noise processes. After transformation, eqn (9) becomes:

$$S_{\mathrm{out},l,p,k}(\omega_1,\omega_2) = H_l(\mathrm{j}\omega_1) H_p(\mathrm{j}\omega_2 - \mathrm{j}\omega_1) S_{l,p,k}(\omega_1,\omega_2), \tag{10}$$

where all indices refer to the corresponding time-domain functions. This result agrees with the two-dimensional Fourier transformation to two *absolute* time variables given in [10]. Because averaging over the absolute time is equivalent to setting $\omega_2 \equiv 0$, the average output noise PSD $S_{\mathrm{out},k}(\omega)$ can finally be obtained by substitution of $\omega_1 = \omega$, $\omega_2 \equiv 0$ into eqn (10). Combination of all four terms of this type then yields

$$S_{\mathrm{out},k}(\omega) = \left|H_l(\mathrm{j}\omega)\right|^2 S_{l,l,k}(\omega) + \left|H_p(\mathrm{j}\omega)\right|^2 S_{p,p,k}(\omega) \\ + \left[H_l(\mathrm{j}\omega) H_p(-\mathrm{j}\omega) + H_l(-\mathrm{j}\omega) H_p(\mathrm{j}\omega)\right] S_{l,p,k}(\omega), \tag{11}$$

where $S_{\mathrm{out},k}(\omega)$ and $S_{l,p,k}(\omega)$ are equivalent to $S_{\mathrm{out},k}(\omega_1, 0)$ and $S_{l,p,k}(\omega_1, 0)$ respectively.

From eqn (11) can be concluded that besides the transfers from the input of state l and state p to the filter output, also their cross-products contribute to the output noise PSD. This was not the case for the type of noise encountered in first-order TL filters. If a noise source contributes to three or more states, the same procedure should be applied to all possible pairs of states.

[1] Thus, in the calculation of $R_x(t,\tau|I_{\mathrm{in}})$, an expectation operation over all large-signal collector currents has been omitted.

V. EXAMPLE

This section demonstrates the proposed analysis procedure for a second-order TL low-pass filter, after [11], depicted in Fig. 3. When $C_1 = C_2$, the filter transfer has a quality factor $Q = 2$. The dashed, fictitious transconductors

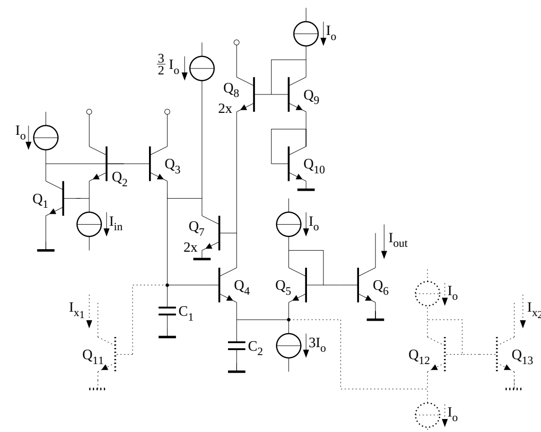

Fig. 3. Second-order low-pass filter after [11].

constructed from Q_{11}-Q_{13} are included to simplify the analysis only: they provide an efficient way to find an equation for the state-variables associated to C_1 and C_2, but neither affect the circuit operation nor produce noise. Further, all transistor (noise) currents, not shown for clarity of the figure, are referred to by the index of the associated transistor, i.e. I_k and i_k for Q_k.

The TL loop equations of the four fundamental DTL loops in the circuit, $Q_1-Q_2-Q_3-Q_{11}$, $Q_{11}-Q_4-Q_{12}-Q_{13}$, Q_7-Q_{10} and $Q_6 - Q_5 - Q_{12} - Q_{13}$, can be written as:

$$(I_\mathrm{o} + i_1)(I_{\mathrm{in}} + i_2) = \left(I_{\mathrm{C}1} + I_7 - \frac{3}{2} I_\mathrm{o} + i_3\right) I_{x_1}, \tag{12a}$$

$$I_{x_1} I_\mathrm{o} = (I_{\mathrm{C}2} + 2I_\mathrm{o} + i_4) I_{x_2}, \tag{12b}$$

$$(I_7 + i_7)(I_{\mathrm{C}2} + 2I_\mathrm{o} + i_8) = 4(I_\mathrm{o} + i_9)(I_\mathrm{o} + i_{10}), \tag{12c}$$

$$(I_{\mathrm{out}} + i_6) I_\mathrm{o} = (I_\mathrm{o} + i_5) I_{x_2}. \tag{12d}$$

According to the DTL principle, the capacitor currents I_{C1} and I_{C2} can be expressed in terms of the state-variables I_{x_1} and I_{x_2} as:

$$I_{C1} = C_1 U_T \frac{\dot{I}_{x_1}}{I_{x_1}}, \qquad I_{C2} = C_2 U_T \frac{\dot{I}_{x_2}}{I_{x_2}}. \tag{13}$$

Then, after simplification w.r.t. the noise sources i_1-i_{10}, the

SSD can be written as:

$$\frac{C_1 U_T}{I_o}\dot{I}_{x_1} = \frac{3}{2}I_{x_1} - 4I_{x_2} + I_{in} + \frac{i_1}{I_o} + i_2 - i_3\frac{I_{x_1}}{I_o} + (i_8 - i_4)4\frac{I_{x_2}^2}{I_o I_{x_1}} + i_7\frac{I_{x_1}}{I_o} - 4\frac{I_{x_2}}{I_o}(i_9 + i_{10}), \tag{14a}$$

$$\frac{C_2 U_T}{I_o}\dot{I}_{x_2} = I_{x_1} - 2I_{x_2} - i_4\frac{I_{x_2}}{I_o} \tag{14b}$$

$$I_{out} = I_{x_2} + i_5\frac{I_{x_2}}{I_o} - i_6. \tag{14c}$$

Observe from this SSD that i_4 is the only 'higher-order TL filter noise' source, that interacts with both filter states. This can be explained from Fig. 3 by the fact that the corresponding transconductor, Q_4, is connected *between* both capacitors C_1, C_2. All other noise sources interact with only one filter state.

The output noise PSD due to i_4 of transistor Q_4, $S_{out,4}(\omega)$, has to be determined from eqn (11). This is done as follows. For brevity, we assume i_4 to represent the collector shot noise of Q_4 only, and the input signal to be the sinusoidal wave $I_{in} = I_{DC}[1 + m\sin(\omega_o t)]$. First the average PSD $S_{l,p,4}(\omega)$ of the noise terms $g_{l,4}(t)i_4(t)$, $l \in \{1,2\}$ and their cross-correlation have to be determined. As observed from the SSD, $g_{1,4}(t) = -4I_{x_2}^2/(I_o I_{x_1})$, $g_{2,4}(t) = -I_{x_2}/I_o$, where I_{x_2} and I_{x_1} should be determined from the noise-free SSD. According to eqn (9), the conditional autocorrelation $R_4(t,\tau|I_{in})$ of i_4 has to be obtained. For non-stationary shot noise, it is known that $R_4(t,\tau|I_{in}) = qI_4(t)\delta(\tau)$, where q denotes the unit-charge and $\delta(.)$ a Dirac-impulse. Then, for this correlation function, application of the two-dimensional Fourier transform to the expectation in eqn (9), and setting $\omega_1 = \omega$, $\omega_2 \equiv 0$ (time-averaging) yields:

$$S_{l,p,4}(\omega) = q\overline{g_{l,4}(t)g_{p,4}(t)I_4(t)} = q\overline{\frac{g_{l,4}(t)g_{p,4}(t)I_o I_{x_1}(t)}{I_{x_2}(t)}}. \tag{15}$$

By substitution of these power density spectra into eqn (11), we can finally determine $S_{out,4}(\omega)$. If $H_1(j\omega)$ denotes the transfer from the input of the first state-variable to the output, i.e. from I_{in} to I_{out}, and $H_2(j\omega)$ the transfer from the second state-variable to I_{out}, $S_{out,4}(\omega)$ can be expressed as:

$$S_{out,4}(\omega) = |H_1(j\omega)|^2\, 16q\overline{\frac{I_{x_2}^3}{I_o I_{x_1}}} + |H_2(j\omega)|^2\, q\overline{\frac{I_{x_1}I_{x_2}}{I_o}} \big[H_1(j\omega)H_2(-j\omega) + H_1(-j\omega)H_2(j\omega)\big]4q\overline{\frac{I_{x_2}^2}{I_o}}. \tag{16}$$

The contributions of all other noise sources to the output noise PSD can be obtained in a similar way, except that eqn (6) should be used instead of eqn (11). The resulting normalized noise PSD for i_4, $S_{out,4}(\omega)/S_{out,4}(0)$, is depicted in figure 4 for $I_{DC} = I_o = 1\mu$A, $m = 0.44$, $C_1 = C_2 = 10$pF, $U_T = 26$mV and $f_o = (I_o/2\pi C_1 U_T) = 61$kHz. For reference, Fig. 4 also depicts the errorneous result for $S_{out,4}(\omega)$ obtained by straight-forward extension of first-order analysis techniques, using eqn (6) instead of eqn (11), which neglects the cross-correlation between the contributions of i_4 to both state-variables. The figure clearly shows that this neglection results in a PSD that is 10 dB too large, corresponding to an overestimation of the output noise power due to i_4 of nearly 7dB.

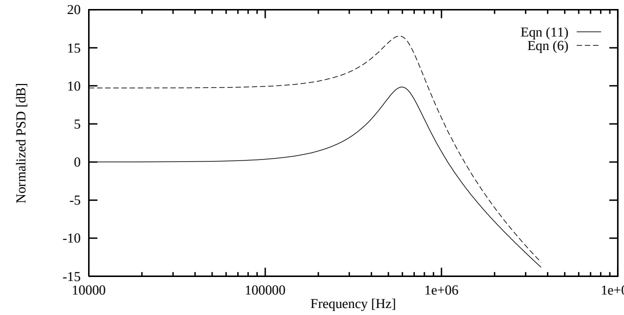

Fig. 4. Normalized output noise PSD $S_{out,4}(\omega)$ calculated according to the new analysis, i.e. eqn (11), and straight-forward extension of first order techniques, i.e. eqn (6).

VI. CONCLUSIONS

An analysis of noise in higher-order TL filters cannot be accomplished by straight-forward extension of techniques for first-order TL filters, due to the presence of 'higher-order TL filter noise', that is not encountered in first-order TL filters. Such noise, affecting multiple filter states in a nonlinear and non-stationary way, requires a new analysis technique.

This paper presented an analysis technique for higher-order TL filters, that is accounts for 'higher-order TL filter noise'. An example analysis for a second-order TL low-pass filter showed that a straight-forward extension of first-order techniques overestimated the output power spectral density and the output noise power due to this type of noise by 10 dB and 7 dB respectively.

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