LOG-DOMAIN WAVELET BASES

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ABSTRACT

A novel procedure to approximate Wavelet bases using analog circuitry is presented. First, an approximation is introduced to calculate the transfer function of the filter, whose impulse response is the required Wavelet. Next, for low-power low-voltage applications, we optimize dynamic range, minimize sensitivity and fulfill sparsity requirements. The filter design that follows is based on an orthonormal ladder structure with log-domain integrators as main building blocks. Simulations demonstrate an excellent approximation of the required Wavelet base (i.e. Morlet). The circuit operates from a 1.2-V supply and a bias current of 1.2μA.

Keywords - Wavelet transform, log-domain filters, orthonormal ladder, analog electronics

1. INTRODUCTION

For signal processing, the Wavelet Transform (WT) has been shown to be a very promising mathematical tool, particularly for local analysis of nonstationary and fast transient signals, due to its good estimation of time and frequency localizations. The Wavelet analysis is performed using a prototype function called Wavelet base, which decomposes a signal into components appearing at different scales (or resolutions). Often systems employing the WT are implemented using Digital Signal Processing (DSP). However, in ultra low-power applications such as biomedical implantable devices, it is not suitable to implement the WT by means of digital circuitry due to the high power consumption associated with the required A/D converter. In [1] we proposed a method for implementing the WT in an analog way. However, besides the derivatives of the gaussian wavelet presented in [1], there are several families of wavelets that have proven to be especially useful [2]. Therefore, a more general procedure to obtain various types of Wavelet bases, is presented in this paper.

Section 2 deals with the computation of a transfer function, which describes a certain Wavelet base that can be implemented as an analog filter. Next, Section 3 describes the complete filter design, taking into account the requirements for low-power low-voltage applications. Some results provided by simulations are given in Section 4. Finally, Section 5 presents the conclusions.

2. WAVELET BASES APPROXIMATION

The flowchart as seen in Fig.1, describes a procedure which generates a transfer function of a wavelet base. The goal of this approach is to be able to reduce the order of the filter without really affecting the approximation of its impulse response. The starting point is the definition of a expression in the time domain which represents the wavelet under investigation. If the wavelet base does not have an explicit expression (e.g., Daubechies wavelets), then the splines interpolation method is used. Subsequently, one determines the appropriate envelope to set the width of the wavelet. Once again, if the envelope does not have an explicit expression, the splines interpolation is applied. In this paper, the Gaussian pulse was chosen as the envelope, which is perfectly local in both time and frequency domains. Once the envelope has been defined, the Padé approximation is executed to find a stable and rational transfer function which is suitable for implementation as an analog filter. The main advantage of the Padé method is its computational simplicity and its general applicability [3]. Therefore, it can easily be applied to other envelopes as well. The Padé approximation is preceded by a two step procedure. First, a Laplace transform is executed and then a Taylor expansion is performed on the expression of the envelope in the Laplace domain. Finally, the wavelet is decomposed into a Fourier series to find the dominant term (the term with the largest coefficient) such that when multiplied with the envelope in the time domain, it results in the approximated wavelet base. The results obtained from the use of this method are illustrated in Fig. 2, where the Morlet and the Daubechies5 (db5) Wavelet bases have been approximated, respectively. Other wavelet bases can also be approximated in a similar manner. The rest of the discussion in this paper shall relate to the design of a Morlet Wavelet filter. In the next section we will map the transfer function onto a state space description that is suitable for low-power implementation.

3. FILTER DESIGN

There are many possible state space descriptions for a circuit that implements a certain transfer function. The same holds for practical realizations. This yields the possibility to find a circuit that fits to the specific requirements of the designer. In the context of low-power, low-voltage analogue integrated circuits, the most important requirements are the dynamic range, the sensitivity and the sparsity, all of which will be treated in the subsections that follow. Moreover, we focus on a synthesis technique, exclusively based on integrators.

3.1. Dynamic Range

A system's dynamic range is essentially determined by the maximum processable signal magnitude and the internally generated noise. It is well known that the system's controllability and observability gramians play a key role in the determination and optimization of the dynamic range. The controllability (K) and observability (W) gramians are derived from the state space description

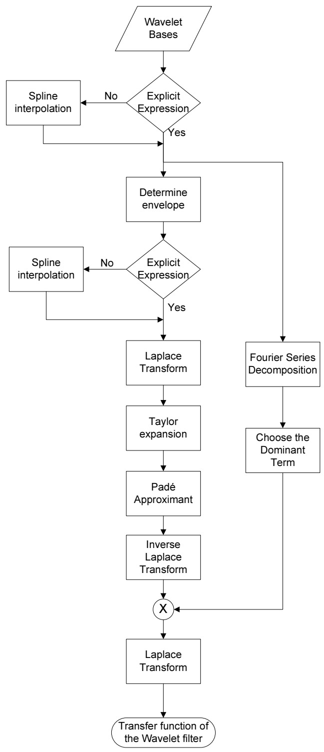


Fig. 1. Flowchart of the Wavelet filters approach

and are computed by solving the equivalent Lyapunov equations

$$AK + KA^T + 2\pi BB^T = 0 \tag{1}$$

$$A^T W + WA + 2\pi C^T C = 0 \tag{2}$$

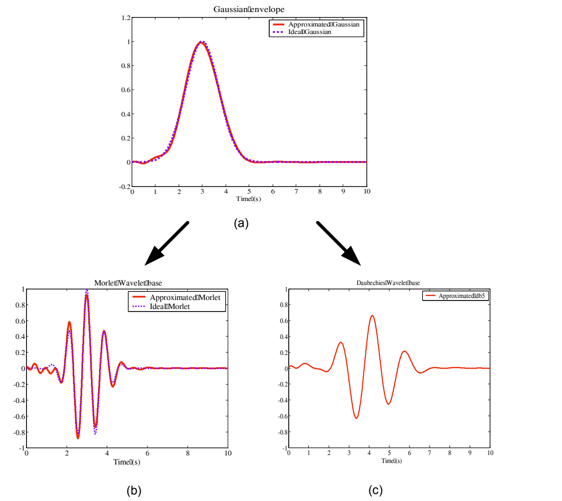
Where A, B and C are the state, input and output matrices of the state-space description, respectively. The entries of A, B and C are derived directly from the coefficients of the transfer function. Optimization of the dynamic range is equivalent to the simultaneous maximization of the (distortionless) output swing and the minimization of the overall noise contribution. The maximum output swing is maximized when the controllability gramian is a diagonal matrix with equal diagonal entries (state scaling). For the minimization of the noise, the observability gramian should become a diagonal matrix as well.

In [4] it is shown that, in order to maximize the dynamic range of the system, one should minimize the objective functional, which represents the relative improvement of the dynamic range and contains all parameters which are subject to manipulation by the designer. The objective functional is given by

$$F_{DR} = \frac{max_i k_{ii}}{(2\pi)^2} \sum_i \frac{\alpha_i}{C_i} w_{ii} \tag{3}$$

where k_{ii} and w_{ii} are the main diagonal elements of K and W respectively, $\alpha_i = \sum_j |A_{ij}|$ is the absolute sum of the elements on the i-th row of A and C_i is the capacitance in integrator i.

Finally, the optimal capacitance distribution is matched to the noise contributions of each individual integrator (noise scaling),



$$H(s) = \frac{0{,}93\,s^8 - 13{,}05\,s^7 + 176{,}91\,s^6 - 617{,}88\,s^5 + 344{,}82 s^4 + 7{,}51\cdot 10^4 s^3 - 4{,}38\cdot 10^5 s^2 + 2{,}44\cdot 10^6 s - 3{,}78\cdot 10^6}{s^{10} + 13{,}3\,s^9 + 336{,}6\,s^8 + 3{,}02\cdot 10^3 s^7 + 3{,}91\cdot 10^4 s^6 + 2{,}40\cdot 10^5 s^5 + 1{,}99\cdot 10^6 s^4 + 7{,}9\cdot 10^6 s^3 + 4{,}40\cdot 10^7 s^2 + 9{,}01\cdot 10^7 s + 3{,}27\cdot 10^8}$$

(d)

Fig. 2. Impulse response of the Wavelet filters, the ideal impulse (dashed line) and the approximated impulse (solid line),(a) Gaussian envelope, (b) Morlet and (c) db5 wavelet base; (d) Transfer function of the Morlet Wavelet filer

i.e, the diagonal entries of W, combined with the coefficients in matrix A, and is defined by [4]

$$C_i = \frac{\sqrt{\alpha_i w_{ii} k_{ii}}}{\sum_j \sqrt{\alpha_j w_{jj} k_{jj}}} \tag{4}$$

Applying the optimization method described in [4] for the transfer function given in Section 2, we have F_{DR} equal to 96.98 which is the absolute minimum value of the objective functional.

3.2. Sparsity

The drawback of a dynamic-range-optimal system is that the state-space matrices is generally fully dense, i.e all the entries of the A, B, C matrices are filled with nonzero elements. These coefficients will have to be mapped onto circuit components, and will result in a complex circuit with a large number of interconnections. For high-order filters it is therefore necessary to investigate how a realization of the desired transfer function having sparser state-space matrices would compare to the one having maximal dynamic range. For a less complex circuit, it is possible, for instance, to reduce A to upper triangular by a Schur decomposition, and by this reducing the number of non-zero coefficients in A [4]. However this transformation leads to an increase in the system noise and consequently to an increase in the objective functional in (3). Another possibility is the Orthonormal Ladder structure [5], which is significantly sparser than the fully dense A matrix of the dynamic-range-optimal system and the Schur decomposition. The advantage of using this structure is its low sensitivity to coefficient (and thus component) mismatch and it will be described in the next section.

3.3. Orthonormal Ladder Structure

Synthesis of analog filters can be achieved in many different ways. However, it is very desirable for the design of high-order filters to concentrate on circuits that give lower sensitivity to component variations. It is known that an optimal dynamic range system will also be optimal with respect to sensitivity. Nevertheless, in order to improve the state-space matrices' sparsity, an orthonormal ladder structure will be implemented which still presents a good behavior with respect to sensitivity. Fig.3 shows a block diagram of a general orthonormal ladder filter [5]. As shown in the block diagram, the filter output y is obtained from a linear combination of the outputs of all integrators x_i.

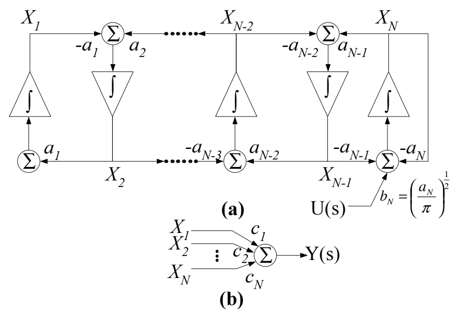


Fig. 3. Block diagram of an orthonormal ladder filter, (a) Leapfrog structure; (b) Output summing stage

The A, B and C matrices of this structure for the defined transfer function is given by

$$A = \begin{bmatrix} 0 & 6.54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.54 & 0 & 1.83 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.83 & 0 & 6.59 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6.59 & 0 & 2.72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.72 & 0 & 6.37 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.37 & 0 & 3.89 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.89 & 0 & 6.27 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -6.27 & 0 & 5.88 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5.88 & 0 & 10.47 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10.47 & -13.31 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2.05 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.75 & -1.34 & 0.75 & 0.68 & -0.57 & 0.44 & -0.002 & -0.10 & 0.04 & 0 \end{bmatrix} \quad (5)$$

The A matrix is tridiagonal and is very nearly skew-symmetric except for a single nonzero diagonal element. The B vector consists of all zeros except for the Nth element. Another property of orthonormal ladder filters is the fact that the resulting circuits are inherently state scaled, i.e., the controllability gramian is already a identity matrix. The drawback of this structure is that the system is not optimized with respect to its noise contribution. However, if an optimal capacitance distribution is applied to this suboptimal system, we still can get some extra gain compared to the case of equal capacitances. Then, the objective functional becomes in that case, $F_{DR} = 147.90$ which is not so far from the optimum case. The Dynamic Range has decreased by only 1.83dB. Finally, the normalized capacitance distribution is given by $(C_1, ..., C_{10}) = C'(0.142, 0.162, 0.110, 0.117, 0.086, 0.091, 0.073, 0.080, 0.073, 0.061)$, where C' represents the unit-less value of the total capacitance when expressed in F.

3.4. Low-power log-domain integrator

The trend toward lower power consumption, lower supply voltage and higher frequency operation has increased the interest of new design techniques for analogue integrated filters. The class of translinear (TL) filters, also known as log-domain filters, has emerged in recent years as a promising approach to face these challenges. The translinear approach is inherently companding and exploits the exponential large-signal transfer function of the semiconductor devices to implement a desired linear or nonlinear differential equation. A simple bipolar multiple-input low-power log-domain integrator [6] will be used as the basic building block for the implementation of the state space equation of a wavelet filter described in previous section. This log-domain integrator is shown in Fig.4. A pair of log-domain cells with opposite polarities and an integrating capacitor form the core of the integrator. V_{ip} and V_{in} are the noninverting and inverting input voltages, respectively, and the input currents are I_{ip} and I_{in}, which are superimposed on the dc bias currents. The output voltage V_{out} is given by the voltage across the capacitor. The circuit is composed of two identical log-domains cells, a voltage buffer and a current mirror. The log-domain cells Q_1-Q_2 and Q_3-Q_4 generate the log-domain currents I_{c2} and I_{c4}, respectively. A voltage buffer realized by Q_5-Q_6 is inserted between them. Therefore, the output log-domain voltage V_o at the emitter of Q_2 also appears at the emitter of Q_4. Finally, to achieve a log-domain integrator equation, a current mirror Q_7-Q_8 is employed in order to realize, in conjunction with the buffer, the difference of the two log-domain currents on the capacitor node. The connection from the bases of transistors Q_7 and Q_8 to the collector of Q_4 closes the feedback loop around Q_4 and Q_7. This connection is convenient because it ensures minimizing the overall voltage headroom. The equation that relates the input and output voltages to the current flowing in the integrating capacitor becomes

$$C_i \frac{dV_{out}}{dt} = (I_o + I_{ip}) e^{\frac{V_{ip} - V_{out}}{V_T}} - (I_o + I_{in}) e^{\frac{V_{in} - V_{out}}{V_T}} \quad (6)$$

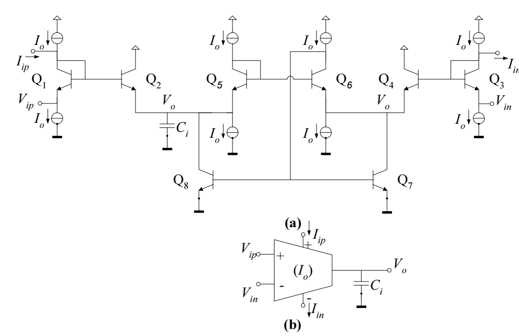


Fig. 4. (a) The multiple-input low power log-domain integrator, and (b) its symbol

Notice that the input and output voltages of the integrator are at the same dc level. Therefore the filter synthesis can be easily achieved by directly coupling these integrators.

3.5. Synthesis of the log-domain state-space filter

By applying a simple mapping to the linear state-space equations (5), the corresponding log-domain circuit realizations can be obtained, using the log-domain integrator cell introduced in the previous section.

The block diagram of the log-domain implementation of (5) is illustrated in Fig.5 using the universal log-domain cell symbol described in [7] and shown in Fig.4b. Note that each column of the filter structure corresponds to a row in the state-space formulation. The parameter A_{ij} is implemented by the corresponding log-domain integrator with bias current $I_{A_{ij}}$, defined by a current matrix A_I

$$A_I = V_T C_i \cdot A \tag{7}$$

The input section, as governed by the state-space vector B, can be defined as the input LOG operator and is realized by the first row from the top of Fig.5. The parameter B is related to the current by

$$B = \frac{I_o}{V_T C_i} \tag{8}$$

Consequently, the B coefficients are not individually controllable by bias currents, and they have to be set equal (or zero). Fortunately, this is the case in (5), where only one non-zero parameter of the B vector is present, and then it is not necessary to transpose the state-space system. Finally, the weighted summation state with the corresponding EXP operators, in order to restore the overall system linearity, should be realized. Then the bias current vector C_I, which controls the vector C, is defined as

$$C_I = I_o \cdot C \tag{9}$$

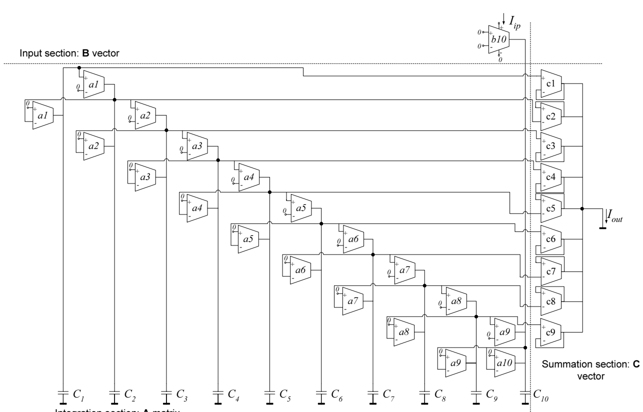


Fig. 5. Complete State-space filter structure

4. SIMULATION RESULTS

To validate the circuit principle, the log-domain state-space filter has been simulated using models of our in-house bipolar semi-custom IC process SIC3A. Typical transistor parameters are $f_{T,max}$ = 15GHz and $\beta_{F,npn}$ = 150 (smallest emitter size). The circuit has been designed to operate from a 1.2V supply and a $100p$F total capacitance. Fig.6 shows the impulse response of the wavelet filter. The total filter's current consumption is 1.2μA. The output current presents an offset of approximately $180p$A.

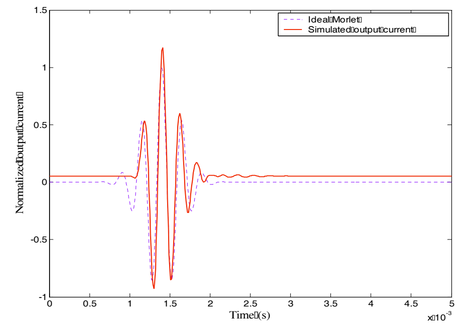


Fig. 6. Simulated impulse response

The excellent approximation of the Morlet can be compared with the ideal Morlet function in Fig.6 to confirm the performance of the log-domain filter.

5. CONCLUSIONS

A novel procedure to approximate Wavelet bases using analog circuitry was presented. Simulations demonstrated an excellent approximation of the Morlet Wavelet base. The circuit operates from a 1.2-V supply and a bias current of 1.2μA. The filter was optimized with respect to dynamic range. Moreover, sensitivity and sparsity were also taken into account when designing the filter. Hence, the filter was able to meet the requirements imposed by a low-power environment. With the results obtained, we deduce that this procedure could very well be used to approximate also other Wavelet bases.

6. REFERENCES

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