MAPPING THE WAVELET TRANSFORM ONTO SILICON: THE DYNAMIC TRANSLINEAR APPROACH

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ABSTRACT

In this paper, an analog implementation of the Wavelet Transform (WT) is presented. The circuit is based on the Dynamic Translinear (DTL) circuit technique and implements, by means of cascade connected complex first order systems, an analog filter whose impulse response is a Gabor function, a function most widely used for frequency analysis among wavelet functions. From simulations, it is demonstrated that we can scale and shift in time and frequency by simply controlling the capacitance or the control current values. The main advantage of this DTL implementation is its low-power consumption. The circuit operates from 1-V supply voltage and a bias current of 1μA.

Keywords – Wavelet transform, Gabor transform, complex first order system, dynamic translinear circuits, ECG characterization, analog electronics

1. INTRODUCTION

For cardiac signal characterization, the Wavelet Transform (WT) has been shown to be a very promising mathematical tool [1]. WT is efficient for local analysis of nonstationary and fast transient signals due to its good estimation of time and frequency localizations. It provides an alternative to the classical Short-Time Fourier Transform or Gabor Transform. It decomposes a signal into components appearing at different scales (or resolutions) [2], using short windows at high frequencies and long windows at low frequencies. As a result, in ECG characterization, wavelet analysis has been used, for instance, to accurately detect the location of the QRS complex, for timing interval measurements, thus evaluating Arrhythmias, and to calculate the entropy of the signal (wavelet time entropy) to detect Myocardial Ischemia.

Unfortunately, for pacemaker applications, it is not favourable to implement the WT by means of digital signal processing because of the high power consumption associated with the required A/D converter.

Because of this constraint, we propose a method for implementing the WT in an analog way by means of dynamic translinear circuits.

Section 2 treats the basic theory of the Wavelet Transform. Next, Section 3 provides an overview of the static translinear (STL) and dynamic translinear (DTL) principles. The circuit design is described in Section 4. Some results provided by simulations are shown in Section 5. Finally, Section 6 presents the conclusions.

2. WAVELET TRANSFORM

Wavelet analysis is a new and promising set of tools and techniques for analyzing non-stationary and fast transient signals. One major advantage afforded by wavelets is the ability to perform local analysis.

The Wavelet Transform is a linear operation that decomposes a signal into components that appear at different scales (or resolutions). The transform is based on the convolution of the signal with a dilated filter, thereby mapping the signal onto a two-dimensional function of time and frequency.

The main idea of the WT is to look at a signal at various windows and analyze it with various resolutions. It provides an alternative to the classical Short-Time Fourier Transform (STFT) or Gabor Transform – Gabor introduced windowed complex sinusoids as basic functions. In contrast to the STFT, which uses a single analysis window, the WT uses short windows at high frequencies and long windows at low frequencies. Thus, the WT is a so-called constant-Q analysis. The Wavelet analysis is performed using a prototype function called mother wavelet, $\psi(t)$ ($\psi(t) \in L^2$, L^2 denoting the Hilbert space of measurable, square-integrable, one-dimensional functions). The main characteristic of the mother wavelet is given by

$$\int_{-\infty}^{\infty} \Psi(t)\,dt = 0 \qquad (1)$$

This means that it is oscillatory and has zero-mean value. Also, this function needs to satisfy the admissibility condition so that the original signal can be reconstructed by the inverse wavelet transform

$$\int_{-\infty}^{\infty} \frac{\left|\widehat{\Psi}(\omega)\right|^2}{|\omega|} = C_\Psi < \infty \tag{2}$$

The admissible condition implies that the Fourier transform of the wavelet must have a zero component at the zero frequency. Hence, the wavelets are inherently band-pass filters in the Fourier domain.

"Any function that has finite energy and is square integrable and satisfies the wavelets admissible condition can be a wavelet" [2].

However, the time-frequency (or time-scale) joint representation has an intrinsic limitation: the product of the resolution in time and in frequency is limited by the uncertainty principle (Heisenberg inequality)

$$\Delta t \Delta \omega \geq \frac{1}{2} \tag{3}$$

Hence, one can only trade time resolution for frequency resolution, or vice versa. Gaussian windows are therefore often used since they meet the bound with equality (minimum time-bandwidth product).

The wavelet transform of a function $f(t)$ at the scale a and position τ is given by:

$$C(\tau, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \Psi^* \left(\frac{t-\tau}{a} \right) dt \tag{4}$$

The factor $1/\sqrt{a}$ is used for energy normalization. The Wavelet transform depends upon two parameters, being scale a and position τ. For smaller values of a, the wavelet transform is contracted in the time domain and gives information about the finer details of the signal. Then, the wavelet becomes more sensitive to high frequency components of the signal. For larger values of the scale a the wavelet is expanded and gives a global view of the signal.

Obviously, the WT is highly redundant when the parameters (a, τ) are continuous. The scale parameter can be sampled along the dyadic sequence $(2^j)_{j\in Z}$,., i.e., $a=2^j$. By imposing that

$$\sum_{j=-\infty}^{\infty} \left|\hat{\Psi}(2^j \omega)\right|^2 = 1 \tag{5}$$

we insure that the whole frequency axis is covered by a dilation of $\hat{\Psi}(\omega)$ by the scales factors $(2^j)_{j\in Z}$.

In many biomedical implantable or injectable applications, the combination of A/D conversion and digital signal processing cannot be done at sufficiently low power consumption. We thus look for an analog implementation of the WT. Moreover, we look for a technique that can be implemented without the need for resistors, since these will become too large at very low current levels. A promising technique, as will be presented below, is the one of dynamic translinear circuits.

3. Static and Dynamic Translinear Principle

Translinear (TL) circuits are based on the exponential relation between voltage and current, characteristic for the bipolar transistor and the MOS transistor in the weak inversion region. They can be divided into Static (STL) and Dynamic Translinear (DTL) circuits.

The STL circuits are implemented to realize any static transfer function. Their principle applies to loops of semiconductor junctions. A TL loop is characterized by an even number of junctions. The number of devices with a clockwise orientation equals the number of counterclockwise oriented devices. An example of a four-transistor TL loop is shown in Fig. 1 [3]. The STL principle states that this circuit can be best described in terms of the collector currents I_1 through I_4. The translinear loop is thus described by a simple equation in terms of products of currents

$$I_1 I_3 = I_2 I_4 \tag{6}$$

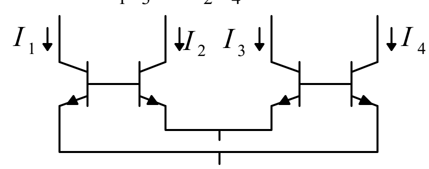


Fig. 1. A four-transistor translinear loop.

Linear or nonlinear dynamic (i.e., frequency-dependent) functions (differential equations) can be implemented by DTL circuits. The DTL principle is shown in the sub-circuit in Fig. 2 [4]. This circuit is described in terms of the collector current I_C and the current I_{cap} flowing through the capacitance C. Note that the dc voltage source V_{const} does not affect I_{cap}. The collector current of a bipolar transistor is based on the exponential law and is described by:

$$I_C = I_s e^{V_{BE}/U_T} \tag{7}$$

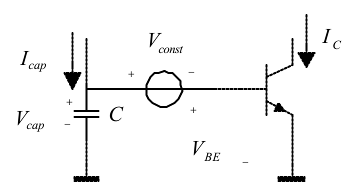


Fig. 2. Principle of dynamic translinear circuits

An expression for I_{cap} can be derived from the time derivative of the collector current and yields

$$C U_T \dot{I}_C = I_{cap} I_C \tag{8}$$

where the dot represents differentiation with respect to time.

This expression defines the principle of dynamic translinear circuits: *"A time derivative of a current is equivalent to a product of currents."*

However, the DTL principle is not limited to the realization of linear differential equations, e.g., filters, and thus it can also be used to implement non-linear differential equations, e.g., oscillators [5] and RMS-DC converters [6].

4. CIRCUIT DESIGN

We will now apply the DTL circuit technique to the design of an analog implementation of the WT.

We first propose an analog Gabor transform filter, of which the impulse response is an approximated Gaussian window function. This analog Gabor Transform filter is implemented with Complex First Order Systems (CFOS) [7]. A CFOS is defined by the following set of equations:

$$\dot{x}(t) = (\sigma_o + j\omega)x(t) + (c_r + jc_i)u(t) \qquad (9)$$

$$x(t) = x_r(t) + jx_i(t) \qquad (10)$$

where u is an input signal assumed to be real, x is a state variable assumed to be complex, σ_o, ω, c_r and c_i are system parameters assumed to be non-positive, positive, real and real, respectively.

After substitution of Eq. (10) into Eq. (9), the real and imaginary part of x, x_r and x_i, can be described by

$$\dot{x}_r = \sigma_o x_r - \omega x_i + c_r u \qquad (11)$$

$$\dot{x}_i = \sigma_o x_i + \omega x_r + c_i u \qquad (12)$$

Next, this set of two real first-order differential equations is realized by means of a DTL circuit.

In Tab. 1, the equivalent dynamic translinear circuits for analog realization of a CFOS are depicted, for complex input signals.

From Eq. (9), we can represent the impulse response of the real input circuit in Tab.1 by the following equation

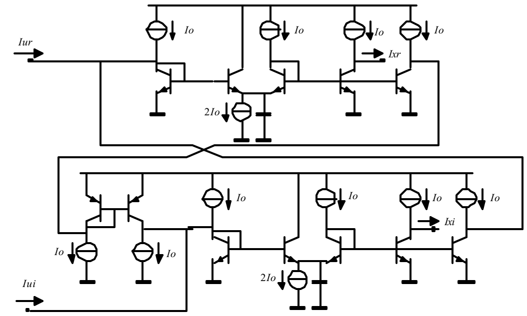
$$h(t) = (c_r + jc_i)e^{\sigma_0 t}U_{-1}(t) \qquad (13)$$

Subsequently, we can connect CFOS's in cascade as shown in Fig. 3 in order to make a sufficient approximation to a Gaussian function.

The impulse response of these (n + 1) DTL/CFOS stages connected in cascade is given by

$$h(t) = (c_r + jc_i)^{n+1}\frac{t^n}{n!}e^{\sigma_0 t}U_{-1}(t) \qquad (14)$$

Note that by increasing the number of stages, we can achieve an increasingly better approximation to the Gaussian function.



(a)

$$\dot{I}_{xr} = \frac{Io}{CU_T}I_{xr} - \frac{Io}{CU_T}I_{xi} + \frac{Io}{CU_T}I_{ur}$$

$$\dot{I}_{xi} = \frac{Io}{CU_T}I_{xi} + \frac{Io}{CU_T}I_{xr} + \frac{Io}{CU_T}I_{ui}$$

(b)

Tab. 1. Equivalent DTL circuit for complex input for the analog CFOS stages in [7]. (a) DTL circuit (b) Differential equations

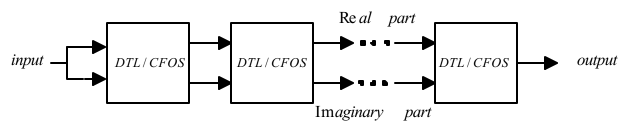


Fig. 3. Cascade connection of DTL/CFOS

As described above, the Wavelet Transform can be obtained by just scaling a (stretching or compressing) and by shifting τ (delaying or hastening) in the Gabor Transform window. Wavelet analysis does not use a time-frequency region, but rather a time-scale region. There are several families of wavelets that have proven to be especially useful. However, for our interest in cardiac signal (one-dimension signal) characterization we use the first derivative of a Gaussian smoothing function, since by doing so the zero-crossings of the wavelet transform indicate the location of the signal sharper variation points.

In the resulting circuit we can scale in time (change a in Eq. 4) by simply controlling the capacitance value C, or, alternatively, the control current, I_o, in the DTL stages.

5. SIMULATION RESULTS

To validate the circuit principle, the circuit was simulated using PSPICE default models. The circuit has been designed to operate from a 1-V supply voltage and a bias current I_o of 1 μA.

The impulse response of the circuit was simulated by applying a unit step waveform in the input and calculating the derivative of the output signal. Fig. 4 shows the resulting responses as a function of the number of stages in our design. As we can see, an improvement in the approximation to a Gaussian is obtained for an increase in the number of stages. This is also verified in Tab. 2, where Δt, $\Delta\omega$ and their product $\Delta t\Delta\omega$ have been given for a cascade of n stages. However, this improvement will be at the expense of a larger noise contribution, or alternatively, a larger current consumption to overcome to effects of accumulation of noise. In general, for the same dynamic range, we need approximately n^2 times the power.

For ECG characterization, we are interested in a first derivative Gaussian wavelet function. In Fig. 5, we can see the Gaussian function and its first derivative for a circuit with 11 stages.

Finally, to implement a Wavelet Transform, we need to be able to scale and shift in time the Gaussian function. By changing the values of the capacitances accordingly we implemented short windows at high frequencies and long windows at low frequencies. See Fig. 6. Alternatively, it is possible to change the value of current I_o, for the resolution in time is proportional to C/I_o.

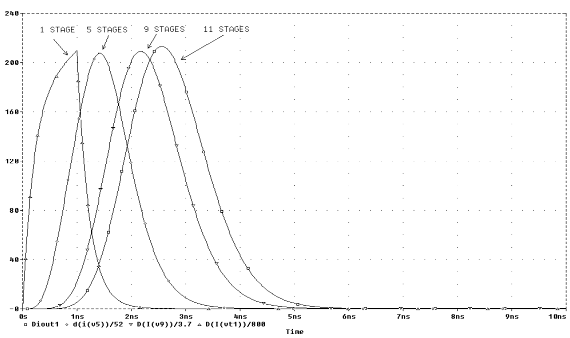


Fig. 4. Impulse response for different number of stages in the connected circuit.

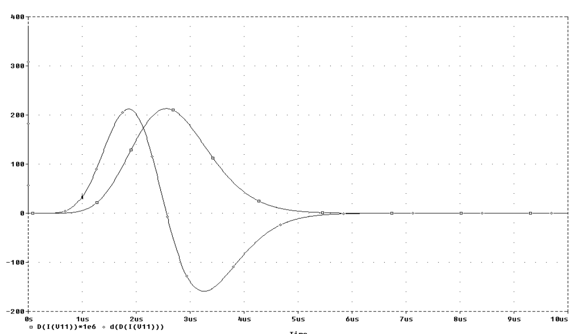


Fig. 5. Gaussian function and its first derivative

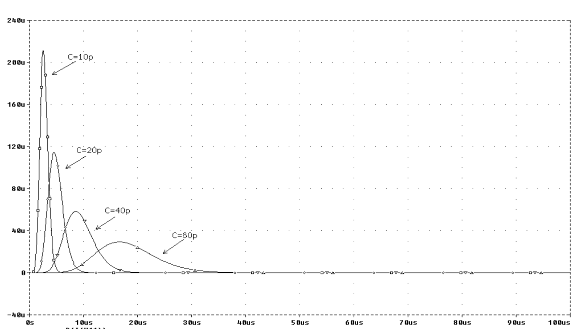


Fig. 6. Gaussian function with different scales

n	Δt	$\Delta\omega$	$\Delta t\Delta\omega$
1	0.7068	1.3732	0.9705
2	0.6124	1.1544	0.7069
3	0.5773	1.0954	0.6323
5	0.5477	1.0541	0.5773
11	0.5222	1.0235	0.5344
50	0.5050	1.005	0.5075
Gaussian	0.5	1	0.5

Tab. 2. Number of stages versus time-bandwidth product

6. CONCLUSIONS

A method for implementing the Wavelet Transform in an analog way by means of dynamic translinear circuits has been proposed. To achieve a sufficient approximation of the impulse response to the desired Gaussian window function, we first proposed an analog Gabor Transform filter by means of a cascade connection of complex first order systems and subsequently implemented these by means of dynamic translinear circuits. Simulations indicate that by increasing the number of stages, we can indeed achieve an increasingly better approximation to the desired Gaussian function. Scaling and shifting in the Gabor Transform window corresponds with controlling the capacitance value or the control current in the DTL stages and yields the desired wavelet transform. The resulting circuit operates from a 1-V supply voltage and a bias current of 1μA. From this low-power consumption, the advantages of analog and DTL over analog-to-digital and digital signal processing become apparent for implantable and injectable biomedical applications. The subject of signal characterization by DTL and analog WT will be the subject of further investigations.

7. REFERENCES

[1] J.S. Sahambi, S.N. Tandon and R.K.P. Bhatt, "Using Wavelet Transform for ECG Characterization," *IEEE Eng. in Medicine and Biology*, pp. 77-83, Jan/Feb. 1997.

[2] O. Rioul and M. Vetterli, "Wavelet and Signal Processing," *IEEE Signal Processing Magazine*, pp. 14-38, Oct. 1991.

[3] W. A. Serdijn, M. Broest, J. Mulder, A. C. van der Woerd and A. H. M. van Roermund, "A Low-Voltage Ultra-Low-Power Translinear Integrator for Audio Filter Applications," *IEEE J. Solid-State Circuits*, vol. 32, no. 4, Apr. 1997, pp. 577-582.

[4] J. Mulder, A. C. van der Woerd, W. A. Serdijn, and A. H. M. van Roermund, "Dynamic translinear circuits – An overview", in Proc. *ISIC*, Singapore, Sept. 10-12, 1997, pp.31-38.

[5] W. A. Serdijn, J. Mulder, A. C. van der Woerd, and A. H. M. van Roermund. "Design of wide-tunable translinear second-order oscillators." In Proc. *ISCAS*, vol. 2, 1997, pp. 829-832.

[6] J. Mulder, A. C. van der Woerd, W. A. Serdijn, and A. H. M. van Roermund. "An RMS-DC converter based on the dynamic translinear principle." in Proc. *ESS-CIRC*, 1996, pp. 312-315.

[7] H. Kamada and N. Aoshima, "Analog Gabor Transform Filter with Complex First Order System," in Proc. *SICE*, 1997, pp.925-930.

[8] N. Aoshima, "Analog realization of Complex First Order System and its application to vowel recognition," in Proc. *SICE*, 1995, pp.1239-1244.