ADAPTIVITY FIGURES OF MERIT AND K-RAIL DIAGRAMS –
– COMPREHENSIVE PERFORMANCE CHARACTERIZATION OF
LOW-NOISE AMPLIFIERS AND VOLTAGE-CONTROLLED OSCILLATORS

Aleksandar Tasic, Wouter A. Serdijn and John R. Long

Delft University of Technology, Department of Microelectronics
Electronics Research Laboratory, UbiCom Research Project
Mekelweg 4, 2628 CD Delft, The Netherlands
Phone: +31 (0)15 278 9423 Fax: +31 (0)15 278 5922
E-mail: a.tasic|w.a.serdijn@its.tudelft.nl

ABSTRACT

Generally, analog RF front-end circuits are designed to be functional under the most stringent, fixed, operating conditions, where all the performance parameters are set by crude hardware design in a fixed way. The figures of merit, resulting from such a design approach, related to only one operating condition and to only one design point, are the already existing, standard or fixed figures of merit. However, the fact that the conditions under which the RF circuits are supposed to be functional are not fixed but rather variant, must be taken into account in both the design of front-end circuits and the characterization of their performances. Accordingly, the figures of merit referring not only to one, but to a set of operating conditions and set of design points are named adaptivity figures of merit. For that purpose, K-rail diagrams are introduced in this paper, offering the possibility for an all-round performance characterization of both adaptive voltage-controlled oscillators and adaptive low-noise amplifiers. The introduced K-rail diagrams give an explicit qualitative explanation of all the existing relations and trade-offs among RF front-end circuit performance parameters such as voltage swing, tank conductance, power consumption, phase noise and loop gain of oscillators as well as noise figure, linearity, gain and power consumption of amplifiers.

1. INTRODUCTION

So far, many parameters and figures of merit characterizing the performances of RF front-end circuits have been introduced, but it is nowhere clearly stated how all those parameters and figures trade among each other. Frequently, there is just a rough impression of how voltage-controlled oscillators (VCOs) can trade power consumption for loop gain and phase noise or how low-noise amplifiers (LNAs) can trade power consumption for noise figure, gain and linearity. What is more, if these key parameters are to be set by the communication system in an adaptive way and not by the hardware design, many concepts fail, due to incomplete knowledge of how the change of one parameter is reflected to the others.

Therefore, the existing figures of merit [1] [2], giving insight into the performance of RF circuits that are considered to operate under fixed conditions must be reformulated in order to be useful for the performance estimation of circuits, employing adaptivity

[3] [4] [5] as a new design paradigm. Therefore, fully new figures of merit such as *phase-noise tuning range* [3], *frequency-transconductance tuning range* [4] and *noise-figure tuning range* [5] are introduced in this paper.

All the *adaptivity figures of merit* and various phenomena related to VCOs and LNAs can be qualitatively interpreted by means of the, in this paper, introduced *K-rail* diagrams. Furthermore, the introduced K-rail diagrams show how RF front-end circuits can trade performance, being phase noise and loop gain of oscillators as well as noise figure, linearity and gain of amplifiers, for power consumption in an adaptive way.

The organization of the paper is as follows. The K-rail diagram is introduced in Section 2, on the example of a quasi-tapped bipolar VCO and an inductively-degenerated LNA. The subject of Section 3 is the K-loop diagram, while Section 4 is left for the conclusions.

2. K-RAIL DIAGRAM

In general, *K-rail* diagrams [3] reveal all the existing *trade-offs* among the performance parameters of an analog RF front-end circuit. Basically, there are two types of K-rail diagrams, *K-rails* diagrams [6] and *K-loop* diagrams [4]. However, as the K-loop diagram is actually the most general form of the K-rails diagram, the latter is omitted from the forth-coming description.

In the analysis to come, we will by introducing in parallel the adaptivity figures of merit (AFOM) and K-rail diagrams, of both a quasi-tapped bipolar VCO [6] and an inductively-degenerated LNA [7], show how the diagrams can be used for the interpretation of various phenomena related to the aforementioned RF front-end circuits.

2.1. The oscillator rail

As the railing concept, to be subsequently introduced, refers to the quasi-tapped VCO, shown in Fig. 1, let us first define its main parameters being the effective tank conductance G_{TK}, the quasi-tapping factor n, the transconductance of the active part of the oscillator G_M and the transconductance seen by the LC-tank $G_{M,TK}$

$$G_{TK} = 1/R_P + R_L/(\omega_0 L)^2 + R_C(\omega_0 C)^2 \qquad (1)$$

$$n = 1 + C_A/C_B, \qquad G_M = g_m/2, \qquad G_{M,TK} = G_M/n \qquad (2)$$

$$L_{TOT} = L, \ C_{TOT} = C + C_A C_B/(C_A + C_B), \ \omega_0 = 1/\sqrt{L_{TOT}C_{TOT}} \qquad (3)$$

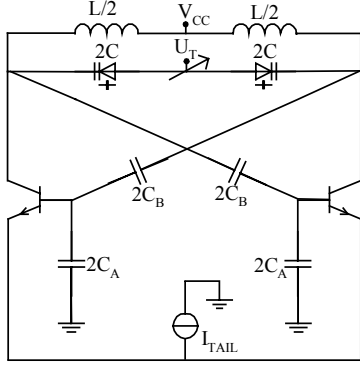Fig. 1 Quasi-tapped LC-oscillator (biasing not shown).

where L is the inductance, C the capacitance, R_L, R_C and R_P their parasitic resistances, g_m the transconductance of the bipolar transistors, U_T the tuning voltage of the varactor C and I_{TAIL} the tail current of the differential pair, being the constant sum of both collector currents.

As suggested by the name of the diagrams (K-rail), the parameter k is given the role of the corner-stone parameter of the analysis. Defined as $k=G_{M,TK}/G_{TK}$ it equally presents the loop gain, the excess conductance seen by the tank as well as the excess power consumption. For the start-up condition it has a value of $k=1$, while for the safe-oscillations the value $k>1$.

Now, we can introduce the *phase-noise tuning range PNTR*, being one of the oscillator's AFOM, that stands for a change in the oscillator's phase-noise between two different biasing (design) points. If $g_{ms-up,QT}$, r_B, and $\mathcal{L}$ are the start-up transconductance of the transistors, the transistors' base resistance and the phase-noise of the oscillator, respectively, the expression for the phase-noise tuning range $PNTR(k_1,k_2)$ of a quasi-tapped VCO, for a k_2/k_1-times increase in power, can be given as [3]:

$$PNTR(k_1,k_2) = \frac{\mathcal{L}_{QT}(k_1)}{\mathcal{L}_{QT}(k_2)} = \frac{k_2^2}{k_1^2}\frac{1+n(k_1/2+c)}{1+n(k_2/2+c)} \quad (4)$$

where c is a positive constant, defined as $c= r_B g_{ms-up,QT}$.

This performance parameter is a direct consequence of the concept of design for adaptivity, that is opting not for a particular operating condition, but rather for a set of conditions. Such phenomena related to whole design space and not its one point only, can qualitatively be described by means of the *K-rail* diagram shown in Fig. 2. Here, it is illustrated how all the oscillator parameters of importance, being loop gain, power consumption, phase noise and signal amplitude, *relate to each other*, in an adaptive manner.

Regarding the use of the K-rail diagram, it should be noted that the arrows in the diagram perpendicular to the corresponding axes, represent the lines of constant loop gain, phase noise and power consumption. Namely, each point in the design space, in this case line (k-rail), corresponds to a set of design parameters, that are actually obtained as a normal projection of the design point on the k-rail to the indicated axes.

For example, if k_{MIN} =2, i.e., the safety start-up condition, and k_{MAX} =6 – expected maximum phase-noise [3], r_B=40Ω and $g_{ms-up,QT}$=8.2mS, the control ranges of the power consumption and the phase noise, both for a quasi-tapping factor n=2, are respectively:

$$P_{MAX}/P_{MIN} = k_{MAX}/k_{MIN} = 3 \quad PNTR(2,6)\,[dB] = 6.3 \quad (5)$$

where P_{MAX} and P_{MIN} represent maximum and minimum power consumption, while $\mathcal{L}_{MAX}$ and $\mathcal{L}_{MIN}$ represent the maximum and minimum phase noise, corresponding to the values of k_{MAX} and k_{MIN}. The phase-noise tuning range is shown in Fig. 2 between the points PN_{MIN} and PN_{MAX}.

In addition, from the *K-rail* diagram, some well known phenomena can easily be recognized. Namely, it is seen that an increase in power results in a certain improvement of the phase noise, but only up to a level determined by k_{MAX}. Increasing the loop-gain beyond this value leads only to a waste of power, as the phase-noise doesn't improve any more.
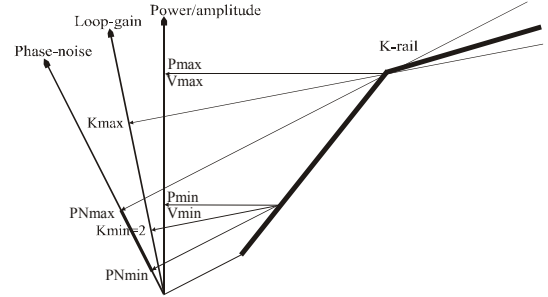

Fig. 2 VCO K-rail diagram.

2.2. The amplifier rail

In the same manner as the K-rail diagram is used for VCOs, it can be used for the explanation of various phenomena related to low-noise amplifiers, as well.

The amplifier topology we will refer to in this sub-section is the inductively degenerated (ID) one [7], shown in Fig. 3.
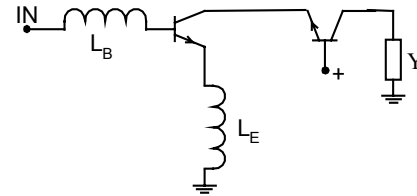

Fig. 3 Inductively-degenerated LNA (biasing not shown).

This is a traditional cascode configuration, where Y stands for the load admittance of the LNA and L_E and L_B stand for the emitter and the base inductors, respectively.

Here, the following parameters are introduced:

$$\text{Re}\{Z_{IN}\} = 2\pi f_T L_E = R_S \quad \text{Im}\{Z_{IN}\} = \omega_0(L_E + L_B) - \frac{\omega_T}{g_m\omega_0} = 0 \quad (6)$$

$$k = r_{EF}g_m(1+1/\beta_F) \quad r_{EF} = r_B + r_E \quad (7)$$

where Z_{IN} is the input impedance, R_S the source resistance, $\omega_0=2\pi f_0$ and $\omega_T=2\pi f_T$ the corresponding operating and transition angular frequencies, g_m the transconductance of the transistors and r_{EF} the equivalent transistor's base and emitter resistance. Yet, B_F and $\beta_F \cong B_F$ are DC and AC transistor current gain factors, respectively.

In this case, assuming that r_{EF} is rather constant and $\beta_F >> 1$, it is obtained that the parameter k is proportional to the biasing condition, i.e., $k \sim g_m$.

Now, the LNA adaptivity figure of merit, the *tuning range* of the *real part* of the *input impedance RITR*, the *voltage-gain*

tuning range VGTR and the *optimum noise-figure tuning range NFTR* [5]*,* defined as a change in the corresponding performance parameter for a certain change in the power consumption $PCR=k/k_{OPT,MIN}$, can be expressed as:

$$RITR(k,k_{OPT,MIN}) = (X_{OPT,MIN} / X - 1)R_S \qquad (7)$$

$$VGTR(k,k_{OPT,MIN}) = X_{OPT,MIN} / X \qquad (8)$$

$$NFTR(k,k_{OPT,MIN}) = \frac{1 + \sqrt{(1+2k)(\beta_F^{-1} + X^2)}}{1 + (1+2k_{OPT,MIN}) / \sqrt{\beta_F(1+k_{OPT,MIN})}} \qquad (9)$$

where the parameter X is defined as $X = \omega_0/\omega_T$, while index $_{OPT,MIN}$ refers to the optimum of the minimum noise-figure.

For example, for $g_{m,OPT,MIN}$=0.27S, β_F=105, r_{EF}=5.2Ω, f_0 =2.4GHz and f_T=29.6GHz, all for *PCR*=1/2 and *PCR*=2, the changes in the real part of the input impedance, the voltage gain and the optimum noise figure, are calculated to be:

$$RITR(0.7,1.4) = -17\Omega \quad RITR(2.8,1.4) = 25\Omega$$

$$VGTR(0.7,1.4)[dB] = -2.2 \quad VGTR(2.8,1.4)[dB] = 1.3$$

$$NFTR(0.7,1.4)[dB] = 0.05 \quad NFTR(2.8,1.4)[dB] = 0.15$$

This phenomenon can also be qualitatively described by means of the *K-rail* diagram, shown in Fig. 4. In this case, the rail of the K-rail diagram represents the design space (design line) of the LNA, with all the construction rules being the same as outlined in the previous sub-section.
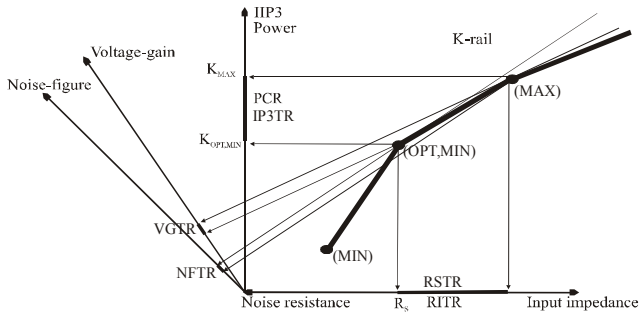

Fig. 4 LNA K-rail diagram.

In addition, from the K-rail diagram, it is seen that an increase in power results in a certain improvement of the noise figure and voltage gain, but only to the levels determined by $k_{OPT,MIN}$ (point OPT,MIN), and k_{MAX} (point MAX), respectively. Also, given certain allowable degradation of the noise figure, considerable power savings (point MIN) as well as linearity improvements (point MAX) are possible, while still keeping the whole RF front-end system within specifications. Note, that all the corresponding tuning ranges are indicated in the diagram, as well.

Summarizing this section let us bring another perspective on the use of the introduced diagrams. Not only are the AFOM and the K-rail diagrams applicable to adaptive circuits, but also they are equally related to circuits not employing adaptivity, circuits where all the performance parameters are set in a fixed way. In such a case, adaptivity is simply considered to be a design margin, most generally, corresponding to the sensitivity of a certain performance parameter to a certain circuit parameter. For example, the AFOM and K-rail diagrams can give one information as to what extent the circuit performances (phase noise, noise figure) would change as a result of a change in biasing condition, i.e., deviation from the required specifications due to an error in the design.

3. K-LOOP DIAGRAM

Since in the case of the K-rail diagrams it was assumed that the LC-tank of the oscillator as well as the degenerative inductance of the amplifier under consideration were fixed, the role of explaining all the phenomena related to the change in both the resonating tank and feedback inductor is left for the *K-loop* diagram. The K-loop diagram is used as a tool for a full *performance characterization* of adaptive RF circuits.

As it has been done in the previous section, in the reminder of the paper we will introduce the oscillator loop as well as the amplifier loop.

3.1. The oscillator loop

Referring to the concept of frequency-transconductance (*C-g_m*) tuning, we will introduce the sensitivity of a tail current (I_{TAIL}) to a tuning voltage (U_T), being the representative of this concept. Showing to what extent the tail current should be changed, as a result of a change in frequency, so as to keep the oscillator operating under the specified conditions, the sensitivity has a form [4]:

$$S_{U_T}^{I_{TAIL}} = -\frac{8k \cdot n \cdot V_T}{a(U_{TO} + \varphi)} \left[R_C \left(1 - \frac{C}{2C_{TOT}} \right) + \frac{C_{TOT}R_L}{2C} \right] (\omega_0 C)^2 \qquad (10)$$

where C_0, φ and a are the parameters of the LC-tank's varactor.

For example, if the oscillator parameters are: f_0=900MHz, $2C$=2pF, Q_C=15, $L/2$=12.5nH, Q_L=4, $2C_A$=1pF, $2C_B$=1pF, φ=0.5V, a=2 and k=2, where Q_C and Q_L are the quality factors of the corresponding varactor and the inductor, then for a maximum tuning voltage range of U_{T0}=1V, the necessary change in the tail current so as to keep the VCO within the specifications is, from Eq. (10), expected to be –0.25mA.

This phenomenon can be qualitatively described by means of the *K-loop* diagram, shown in Fig. 5. Compared to the K-rail diagram, here, one more axis is introduced, referring to the tank conductance and the oscillating frequency. Also, two more rails are added, each corresponding to a differently tuned LC-tank.
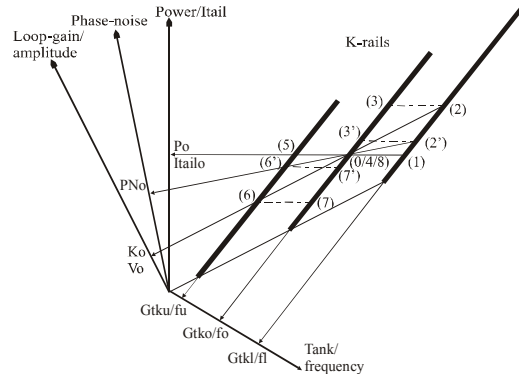

Fig. 5 VCO K-loop diagram.

To explain the use of the K-loop diagram, let us make one loop from point (0) to point (4), both corresponding to the central resonant frequency. As a result of a tuning to a lower frequency, the effective tank conductance becomes larger, which is equivalent to a moving from a position (0) to a position (1), as the inserted power (tail current) is at the same level. It can be noticed that at point (1), all the performance parameters are degraded, which might mean that the required specifications are not satisfied

any more. To compensate for such a deterioration of the performances, the power level (tail current) must be increased by an amount indicated by Eq. (10). This corresponds to the next position on the k-rail, point (2). Now, the loop gain, the amplitude and the phase noise are brought to the level where the specs are met again. Next, moving from point (2) to point (3) is equivalent to a tuning back to the central resonant frequency. As in this point all the performance parameters are at unnecessary higher levels, the tail current can be reduced – point (4), bringing us back to the starting point (0), at the same time offering some power savings.

3.2. The amplifier loop

Following the analogy with the concept of C-gm tuning, let us introduce the "fictitious" concept of *L-gm tuning,* related to ID LNAs, standing for a "change" in the degenerative inductance, as a result of a change in a biasing condition (adaptivity), so as to keep the amplifier operating within the required specifications. However, as tunable integrated inductances are not available at present, by introducing this concept our intention is actually to determine what is the most favorable condition, regarding the chosen feedback inductance, under which the LNA should operate. Accordingly, the sensitivity of the inductance (L_E) to a biasing point (g_m) is, with the aid of Eq. (4), derived to be:

$$S_{g_m}^{L_E} = \partial L_E / \partial g_m \cong -L_E / g_m \tag{11}$$

For example, for the 50Ω power-matching inductance L_E =0.27nH and the optimum transconductance g_m=0.27S, the resulting sensitivity equals -1nH/S, implying that a change in transconductance for 1S, must be followed by a change in inductance for 1nH, so as to always keep the input of the LNA power-matched. This phenomenon can also be qualitatively described by means of the *K-loop* diagram, shown in Fig. 6, where, compared to the diagram of Fig. 4, all the L_E-insensitive axes are omitted.
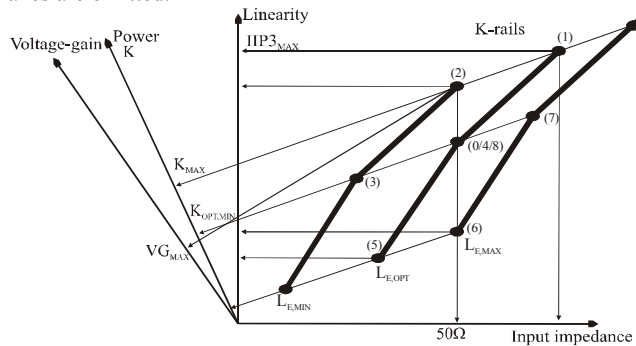

Fig. 6 LNA K-loop diagram.

Note, that two more rails have been added, the left-most one, corresponding to the 50Ω-matching inductance $L_{E,MIN}$ for the maximum power-consumption level – point (2), and the right-most one, corresponding to the 50Ω-matching inductance $L_{E,MAX}$ for the minimum power level – point (6). Both inductance values, $L_{E,MAX}$ and $L_{E,MIN}$, can easily be obtained from Eq. (11).

To explain the use of the diagram let us focus on two characteristic points of the diagram, point (2) and point (6). Compared to point (1), operating in point (2) has both an advantage of higher gain, due to 50Ω-matching (less reflection) and lower amount of feedback, as well as a drawback of, due to lower inductance $L_{E,MIN}$, degraded linearity. This mode of operation is favorable when, for example, the front-end receiving

signal is rather weak, necessitating for higher gain and, at the same time, lower over-all noise figure.

Similar, operating in point (6) is chosen when both the receiving desired signal and the interference signal are stronger, accordingly necessitating for better linearity (third order intercept point *IP3*) and lower gain. Compared to point (5), exactly these performances are found in point (6), where the larger inductance $L_{E,MAX}$, is responsible for higher *IP3* and lower gain. As seen from the diagram, this mode of operation can be denoted to as the "*low-power*" one, unlike the mode related to point (2), being the "*high-power*" one.

Finally, on a journey throughout the *K-loop* diagrams, not only can all the previously addressed phenomena be recognized, but also all the possible trade-offs among the power consumption, phase noise, gain, noise figure and linearity can be qualitatively interpreted.

4. CONCLUSIONS

Referring not only to one, but to a set of possible operating conditions and set of design points, a new approach in both the design and the performance characterization of analog RF front-end circuits is required.

Therefore, by introducing in parallel the adaptivity figures of merit and K-rail diagrams, it has been shown in this paper, how the diagrams can be used for the interpretation of various phenomena related to adaptive low-noise amplifiers and adaptive voltage-controlled oscillators. Also, it has been shown how the K-rail diagrams can be used for qualitative description of all the existing relations and trade-offs among RF circuits' parameters such as voltage-swing, tank conductance, phase noise, gain, noise figure, linearity and power consumption, at the same time giving one the opportunity of controlling all the performances over the whole range of operation.

Finally, the usefulness of the diagrams can easily be foreseen, after one is confronted with the various RF circuits related phenomena depending on a number of parameters that are moreover mutually dependent.

5. REFERENCES

[1] A. Ham and A. Hajimiri, "Concepts and Methods in Optimization of Integrated LC VCOs", *IEEE Journal of Solid-State Circuits,* vol. 36, no. 6, pp. 896-909, June 2001.

[2] M. Soyuer et al. "A 5.8GHz 1V Low-Noise Amplifier in SiGe Bipolar Technology", *Proceedings RFIC1997.*

[3] A. Tasic and W.A. Serdijn, "Concept of Phase-Noise Tuning of Bipolar Voltage-Controlled Oscillators", *Proceedings ISCAS2002.*

[4] A. Tasic and W.A. Serdijn, "Concept of Frequency-Transconductance Tuning of Bipolar Voltage-Controlled Oscillators", *Proceedings ISCAS2002.*

[5] A. Tasic and W.A. Serdijn, "Adaptive Low-Noise Amplifiers", *Proceedings MMSA2002.*

[6] A. Tasic and W. A. Serdijn, "Concept of Quasi-Capacitive Tapping of Bipolar Voltage-Controlled Oscillators", *Proceedings ICECS2002.*

[7] S.P. Voinigescu et al. "A Scalable HF Noise Model for Bipolar Transistors with Application to Optimal Transistor Sizing for LNA Design", *IEEE JSSC*, vol. 32, no. 9, Sep. 1997.