K-Rail Diagrams – Comprehensive Tool for Full Performance Characterization of Voltage-Controlled Oscillators

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ABSTRACT

So far, many parameters and figures of merit characterizing the performances of the voltage-controlled oscillators have been introduced, but it is nowhere clearly stated how all those parameters and figures trade among each other. Frequently, there is just a rough impression of how oscillators can trade power consumption for loop-gain and phase-noise. What is more, if these key parameters are to be set by the communication system in an adaptive way and not by the crude hardware design, many concepts fail, due to incomplete knowledge of how the change of one parameter is reflected to the others. For this purpose, *K-rail* diagrams are introduced in this paper, offering the possibility for an all-round performance characterization of voltage-controlled oscillators, including adaptivity. Also, the *K-rail* diagrams give an explicit qualitative and quantitative explanation of all the existing relations and trade-offs among the oscillator parameters such as voltage-swing, tank conductance, power consumption, phase-noise and loop-gain.

1. INTRODUCTION

Wireless telecommunication transceivers of today are supposed to support high data rates demanded by the applications, to consume as little energy as possible and to adapt themselves to varying channel conditions and user requirements. However, current analog RF front-end circuits are designed to perform only one specific task, setting dynamic range, bandwidth, selectivity and the like by the hardware in a fixed way. As a result, transceiver's topologies that are designed to be functional under the most stringent conditions, suffer from a lot of overhead in both circuit complexity and power consumption. Therefore, the concept of design for adaptivity must be taken into account as the starting point in the design of today's front-end circuits as well as in the characterization of their performances.

Accordingly, the existing figures of merit [1] [2], giving insight into the performance of the circuits, in this case oscillators, that are considered to operate under fixed conditions, must be reformulated in order to be useful for the performance estimation of the adaptive circuits, employing adaptivity [3] [4] as a new design paradigm. On the other hand, fully new figures of merit and new tools are expected to appear as well as it is the case with phase-noise tuning [3], frequency-transconductance tuning [4] and the *K-rail* diagrams, introduced in this paper.

The *K-rail* diagrams, show how the oscillator can trade performance, being phase-noise and loop-gain, for power consumption in an adaptive way. Furthermore, all the parameters and figures of merit, characterizing the oscillator's current state, can be simultaneously followed using a unique presentation in the form of the *K-rail* diagrams, being suitable for both qualitative and quantitative description.

The organization of the paper is as follows. General remarks regarding the diagrams are given in Section 2. The *K-rail* diagram is introduced in Section 3, on the example of a quasi-tapped bipolar voltage-controlled oscillator (VCO). The subject of Section 4 is the *K-rails* diagram, while the *K-loop* diagram is explained in Section 5. An all-round example, showing how all the performance parameters of the oscillator can be mapped onto the corresponding *K-rail* diagrams, is presented in Section 6. Section 7 is left for the conclusions.

2. RAILING CONCEPT

Before introducing the concept of *K-rail* diagrams, let us first see how are they classified, what is the scope of their use, and why is it beneficial to use them.

There are basically two types of *K-rail* diagrams, being *K-rails* and *K-loop* diagrams, both using the same construction rules, as will be explained in next section.

Regarding its use, the *K-rails* diagram is used as a tool for the all-encompassing performance *comparison* of different voltage-controlled oscillators, for the sake of simplicity, in this paper, non-tapped (*NT*) and quasi-tapped (*QT*) VCO's.

On the other hand, the *K-loop* diagram is used for the performance *characterization* of fully adaptive voltage-controlled oscillators. Namely, it is known how all the relevant parameters, phase-noise, loop-gain, power consumption, voltage-swing and tank conductance, are related to each other in any fixed point in the design space as well as it is known to what extent are all those parameters changed in-between any two points of the same design space.

3. K-RAIL DIAGRAM

For the purpose of the analysis to come, we will refer to a bipolar VCO, shown in Fig. 1, as to a quasi-tapped bipolar VCO [5]. The main parameters of the oscillator are defined as:

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$$G_{TK} = \frac{1}{R_P} + \frac{R_L}{(\omega_0 L)^2} + R_C(\omega_0 C)^2 \quad (1)$$

$$n = 1 + \frac{C_A}{C_B}, \quad G_M = g_m/2, \quad G_{M,TK} = G_M/n \quad (2)$$

$$L_{TOT} = L, \; C_{TOT} = C + \frac{C_A C_B}{C_A + C_B}, \; \omega_0 = \frac{1}{\sqrt{L_{TOT} C_{TOT}}} \quad (3)$$

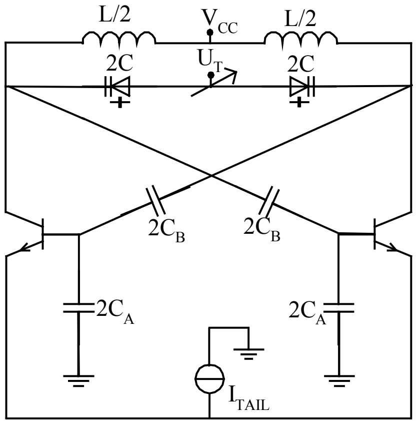

Fig. 1 Quasi-tapped LC-oscillator.

where L is the inductance, C the capacitance, R_L, R_C and R_P their parasitic resistances, G_{TK} the effective tank conductance, C_A and C_B the quasi-tapping capacitances, n is the quasi-tapping factor, G_M the transconductance of the active part of the oscillator, $G_{M,TK}$ the transconductance seen by the LC-tank, g_m the transconductance of the bipolar transistors, U_T the tuning voltage of the varactor C, and I_{TAIL} the tail current of the differential pair, being the constant sum of both collector currents.

The usefulness of the diagrams can easily be foreseen, after one is confronted with the complicated expressions depending on a number of parameters that are moreover mutually dependent.

If $g_{ms\text{-}up,NT}$, r_B, $k=G_{M,TK}/G_{TK}$ and $\mathcal{L}$ are the start-up transconductance of the transistors, the transistors' base resistance, the loop-gain and the phase-noise of the oscillator, respectively, the expression for the phase-noise tuning range *TR(k₁,k₂)* [3] of a quasi-tapped VCO, for a k_2/k_1-times increase in power, is given as:

$$TR(k_1,k_2) = \frac{\mathcal{L}_{QT}(k_1)}{\mathcal{L}_{QT}(k_2)} = \frac{k_2^2}{k_1^2}\frac{1+n(k_1/2+c)}{1+n(k_2/2+c)} \quad (4)$$

where c is a positive constant, defined as $c = r_B g_{ms-up,QT}$.

This performance parameter is direct consequence of the concept of design for adaptivity, that is opting not for a particular operating condition, but rather for a set of conditions. Such a case where designers are dealing with a whole design space and not its one point only, can qualitatively be exposed by means of the *K-rail* diagram shown in Fig. 2. Here, it is illustrated how all the oscillator parameters of importance, being loop-gain, power consumption, phase-noise and signal amplitude, *relate to each other*, in an adaptive manner.

Regarding the use of the *K-rail* diagram, it should be noted that the arrows in the diagram represent the lines of constant loop-gain, phase-noise and power-consumption. Namely, each point in the design space, in this case line (k-rail), corresponds to a set of design parameters, that are actually obtained as a normal projection of the design points on the k-rail to the indicated axes.

For example, if $k_{MIN} = 2$, i.e., the safety start-up condition, and $k_{MAX} = 6$ – expected maximum phase-noise [3], $r_B = 40\Omega$ and $g_{ms\text{-}up,QT} = 8.2mS$, the control ranges of the power consumption and the phase-noise, both for a quasi-tapping factor $n = 2$, are respectively:

$$\frac{P_{MAX}}{P_{MIN}} = \frac{K_{MAX}}{K_{MIN}} = 3$$

$$TR(2,6) = \frac{\mathcal{L}_{QT,MIN}}{\mathcal{L}_{QT,MAX}} = 4.25 \qquad TR(2,6)[dB] = 6.3 \quad (5)$$

where P_{MAX} and P_{MIN} represent maximum and minimum power consumption, while $\mathcal{L}_{\text{MAX}}$ and $\mathcal{L}_{\text{MIN}}$ represent the maximum and minimum phase-noise, corresponding to the values of k_{MAX} and k_{MIN}. The phase-noise tuning range is shown in the Fig. 2 between the points PN_{MIN} and PN_{MAX}.

In addition, from the *K-rail* diagram, some well known phenomena can easily be recognized. Namely, it is seen that an increase in power results in a certain improvement of phase-noise, but only up to a level determined by k_{MAX}. Increasing the loop-gain beyond this value leads only to a waste of power, as the phase-noise doesn't improve any more.

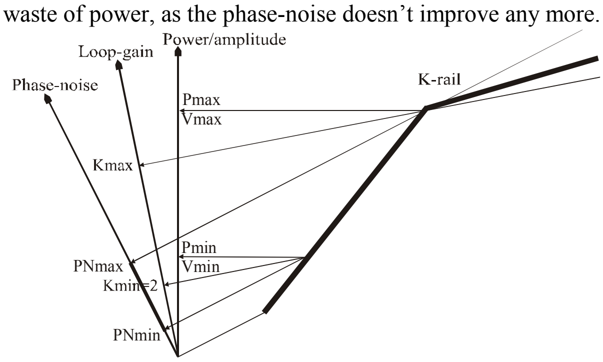

Fig. 2 *K-rail* diagram.

Finally, the presented diagram helps one, without plunging into the "sea" of existing figures of merit, theories and expressions, to grasp all the basic concepts regarding behavior and functionality of the VCO's in an easy to understand manner.

4. K-RAILS DIAGRAM

To have better understanding of the *K-rails* diagrams, let us first introduce the following parameter. Similar to the derivation of the phase-noise tuning range, the ratio between phase-noise of a non-tapped (n=1) and a quasi-tapped oscillator *R(k₁,k₂)* [5], can be defined as:

$$R(k_1,k_2) = \mathcal{L}_{NT}(k_2)/\mathcal{L}_{QT}(k_1)$$

$$R(k_1,k_2) = \frac{2n^2k_1^2}{2-(n-1)\cdot n\cdot k_1 + 2n^2 A_{NT}(k_1)}\frac{1+A_{NT}(k_2)}{k_2^2} \quad (6)$$

where the function A_{NT} is:

$$A_{NT}(k) = k/2 + c\,, \quad c = r_B g_{ms-up,NT} \quad (7)$$

Note that index $_1$ corresponds to a quasi-tapped and index $_2$ to a non-tapped oscillator model.

The operating conditions that are used in the analysis are the one for the same power consumption ($k_1=nk_2$), and the one for the same distance from the start-up condition ($k_1=k_2=k$). Without loss of generality, and for easier interpretation, we will assume that the quasi-tapping factor equals two, i.e., $n=2$.

Now, the phase-noise ratio for the same power consumption equals:

$$R(k_1,2k_1) = \frac{1+A_{NT}(2k_1)}{1-k_1+4A_{NT}(k_1)} = \frac{1+k_1+c}{1+k_1+4c} < 1 \quad (8)$$

As expected, a non-tapped oscillator has always better performance than a quasi-tapped oscillator, with respect to the phase-noise for the same power consumption. For example, if $k_1=1$ – the start-up condition, $r_B=40\Omega$ and $g_{ms\text{-}up,NT}=4.1mS$, there is a difference in phase-noise of 0.8dB in favor of the non-tapped oscillator.

This rather complicated expression can however be qualitatively mapped onto the *K-rails* diagram shown in Fig. 3. Namely, this type of diagram can be used as an all-encompassing tool for the performance *comparison* of non-tapped (*NT*) and quasi-tapped (*QT*) VCO's. It is simply obtained by allocating separate k-rails to each of the oscillators.

Now, following the rules regarding the use of the diagrams, presented in previous section, it can be seen that for the same power consumption $P_1=P_2$, and accordingly $k_2=nk_1$ (points A and B), holds $PN_2>PN_1$, i.e. phase-noise ($\mathcal{L}=1/PN$) of a non-tapped oscillator is better than the one of a quasi-tapped, as already indicated by Eq. (8). Note that the left rail corresponds to the non-tapped and the right rail to the quasi-tapped oscillator.

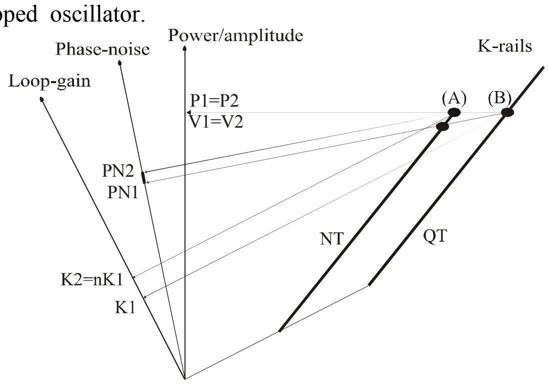

Fig. 3 *K-rails* diagram - the same power consumption.

In a similar manner, a phase-noise ratio, for the same excess negative conductance and $n=2$, is given as:

$$R(k,k)=\frac{4+4A_{NT}(k)}{1-k+4A_{NT}(k)}=\frac{4+2k+4c}{1+k+4c}>1 \qquad (9)$$

showing that a quasi-tapped oscillator has always better performance than a non-tapped oscillator, with respect to the phase-noise for the same loop-gain.

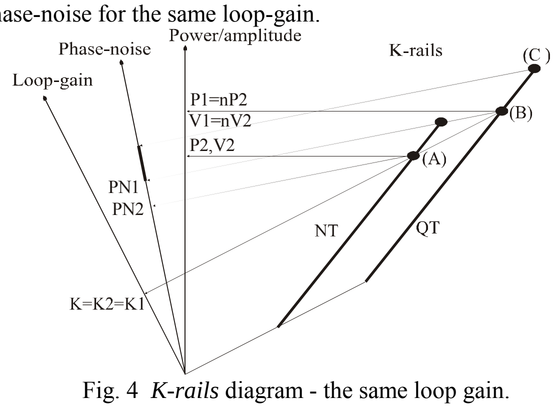

Fig. 4 *K-rails* diagram - the same loop gain.

Using the same example as in the previous case and for $k=k_{MAX}=6$ [3], there is a difference of 3.4dB in phase-noise, in favor of a quasi-tapped oscillator. This is shown in Fig. 4, where for the phase-noise corresponding to the points A and B ($k_2=k_1=k$), holds $PN_2<PN_1$. Also, the region of a superior quasi-tapped VCO performance (B – C) is easily recognized in the diagram.

Finally, we can stress that the presented diagrams serve *qualitative comparison* of performances of differently tapped VCO's. A number of parameters can simultaneously be compared, as it can also be seen to what extent the change in one parameter is reflected to the other parameters.

5. K-LOOP DIAGRAM

Since in the case of *K-rails* diagram it was assumed that the LC-tank of the oscillator under consideration was fixed, the role of explaining all the phenomena related to the change in the LC-tank will be given to the *K-loop* diagram.

Relying on the concept of frequency-transconductance (C-g_m) tuning [4], standing for compensating of the changes in oscillation condition imposed by frequency tuning, we will construct a loop diagram. But first, let us say a few words regarding the phenomenon of C-g_m tuning.

It is well known that as a result of a change in frequency by tuning the varactor (C) of the LC-tank, the loop-gain, the voltage swing and the phase-noise of the oscillator are changed as well. If the oscillator is designed at the very edge of the required specifications, it can be that the change of the oscillation condition, i.e., loop-gain, or the amount of produced noise, i.e., phase-noise, put the oscillator out of operation. Therefore, it is necessary to apply a controlling mechanism, i.e., control of biasing condition (g_m), being able to keep the oscillator still functioning. Accordingly, the concept of C-g_m tuning, mapped onto the *K-loop* diagram, shows what is the relation between the tuning voltage U_T of the LC-tank's varactor, and the tuning biasing current I_{TAIL} of the oscillator's active part. Also, it is shown, to what extent the power consumption and the phase-noise of oscillator are affected by the tuning process.

To what extent the tail current should be changed, as a result of a change in frequency, so as to keep the oscillator operating under the specified conditions, is given as [4]:

$$S_{U_T}^{I_{TAIL}}=-\frac{8k\cdot n\cdot V_T}{a(U_{TO}+\varphi)}\left[R_C\left(1-\frac{C}{2C_{TOT}}\right)+\frac{C_{TOT}R_L}{2C}\right](\omega_0 C)^2 \qquad (10)$$

where C_0, φ and a are the parameters of the varactor.

All the accompanying effects of the change in the tank will be now illustrated by means of *K-loop* diagram. Namely, the diagram shown in Fig. 5 can be used as an all-encompassing tool for the characterization of the performances of any complexity.

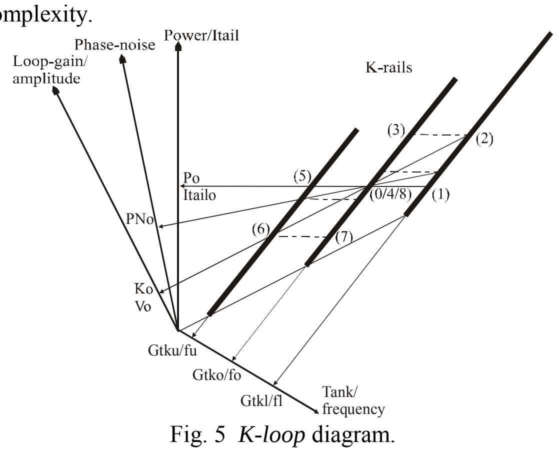

Fig. 5 *K-loop* diagram.

To explain its meaning, let us therefore make one loop from point (0) to point (4) in the diagram, corresponding to the C-g_m tuning for a constant loop-gain. For that purpose, the following should be noted, as well. Indexes $_{0,\ L}$ and $_U$ refer to the central resonant frequency f_0, the lower resonant frequency f_L and the upper resonant frequency f_U, respectively. Also, apart from the axes, corresponding to the phase-noise, loop-gain and power consumption, three, so-called, k-rails are shown as well. They correspond to the LC-tank at the frequencies f_L, f_0 and f_U, as indicated at the tank/frequency axis.

As a result of tuning to a lower frequency, the total capacitance as well as the effective tank conductance become larger, which is equivalent to a moving from a position (0) to a position (1), as the inserted power (tail current) is at the same level. It can be noticed that at point (1), the voltage swing over the resonator, the loop-gain and the phase-noise are decreased, accordingly. To compensate for such a deterioration of the performances, the power level (tail current) must be increased for an amount indicated by Eq. (10). This corresponds to the next position on the k-rail – point (2). As seen, the loop-gain and the amplitude are brought again to the level as being before the tuning action. What is more, the phase-noise is even improved compared to the starting point. Next, the reduction of the capacitance of the varactor and according increase in oscillating frequency correspond to point (3). As the phase-noise, loop-gain, amplitude, and power consumption are at unnecessary higher levels, the tail current can be reduced, so that all the specifications are met again – point (4). Namely, as shown in the diagram, point (4) corresponds to the starting point (0). Similar reasoning holds for the left loop, being the one consisting of points (4) to (8).

On a journey throughout the *K-loop*, not only can all the previously addressed situations be recognized, but also all the possible trade-offs between the power consumption, phase-noise and loop-gain can be qualitatively interpreted.

6. AN ALL-ROUND EXAMPLE

To prove the validity of the introduced concepts as well as to show how powerful tool is the *K-loop* diagram, for both qualitative and quantitative representation of the all addressed phenomena, a complete and fully realistic example is presented. As in the previous section, we will refer to the quasi-tapped bipolar VCO, shown in Fig. 1, in the forthcoming analysis.

The values of the oscillator parameters are as follows: f_0=900MHz, 2C=2pF, Q_C=15, L/2=12.5nH, Q_L=4, 2C_A=1pF, 2C_B=1pF, V_{CC}=2V, U_{T0}=1V, φ=0.5V, a=2, and k=2, where Q_C and Q_L are the quality factors of the corresponding varactor and the inductor. To the case when the loop-gain k equals 2, we will refer as to the safety start-up condition, and denote it with the index $_{S\_S\text{-}UP}$.

To see what are the effects of both voltage tuning and the corresponding C-g_m tuning, we will use the diagram, shown in Fig. 6, as the cornerstone of the analysis. Also, it will be shown how the loop-gain, the tail current, the effective tank conductance and the phase-noise can be found for any point in the diagram.

Therefore, we should first find the values of the tank conductance for the lower, central and upper resonant frequency of the oscillator. From Eq. (1), it is found that the tank conductance G_{TK} equals 2.05mS at a frequency f_0=900MHz, while from Eq. (2) the tail current is found to be $I_{TAIL,S\_S\text{-}UP}$ =0.85mA. For a maximum voltage tuning range of 1V, the maximum change of a tail current is, from Eq. (10), expected to be 0.25mA. This means that in order to sustain oscillations under the same condition, being a loop-gain of two, the tail current should be either increased or reduced by this amount of current. As the order of the points is the same as in Fig. 5, points (2) and (3) of the diagram correspond to a tail current of 1.1mA, while points (6) and (7) correspond to a current of 0.6mA. Now, knowing the loop-gain at position (2) and referring to a tail current of 1.1mA as to the safety start-up current for a new tank, it is rather straightforward to calculate, from Eq. (2), that the new tank conductance $G_{TK,L}$, corresponding to a lower resonant frequency f_L=780MHz, has a value 2.64mS. This is for a tuning range of ±120MHz. The loop-gain for the point (3) is simply found to be k=2.6. The parameters of the left-loop, (4) to (8), are calculated following the same procedure.

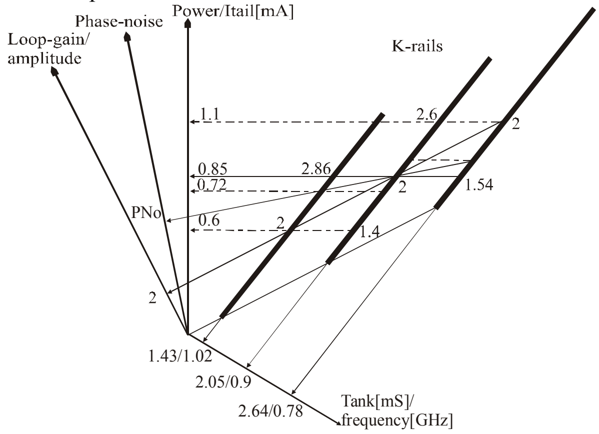

Fig. 6 *K-loop* diagram for the given parameters of the VCO.

Finally, the parameters, such as, the phase-noise ratio R and the phase-noise tuning range TR, can easily be found and allocated to the points of the *K-loop* diagram. Namely, from Eq. (8), the ratio R between points (0) and (2) is estimated to be around 1dB, while from Eq. (4), the range TR between points (0) and (3) is calculated to be around 2dB.

7. CONCLUSIONS

Frequent changes of the conditions under which the communication systems of today operate, necessitate for a new approach in both the design and the performance characterization of analog RF front-end circuits. Accordingly, as a direct result of the concept of structured electronic design for adaptivity, the presented *K-rail* diagrams are considered to be a tool for an all-encompassing performance characterization of adaptive voltage-controlled oscillators.

It has been shown how the *K-rail* diagrams can be used for both qualitative and quantitative description of all the existing relations and trade-offs among the oscillator parameters such as voltage-swing, tank conductance, power consumption, phase-noise and loop-gain, in an easy to grasp manner.

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