Concept of Quasi-Capacitive Tapping of Bipolar Voltage-Controlled Oscillators

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ABSTRACT

Generally, bipolar transistors have better performance than MOS transistors if higher transconductances for the same operating point or higher transition frequencies are required. On the other hand, compared to MOS transistors, bipolar transistors often have limited use in the design of VCO's, due to a limited voltage swing across the resonator and therefore limited phase-noise performance. However, applying quasi-capacitive tapping between the active part of the oscillator and the LC-tank, as proposed in this paper, the voltage swing over the tank can be increased accordingly, while at the same time the transistor remains far from heavy saturation. The presented analytical expressions give an insight into the trade-offs between quasi-tapped and non-tapped oscillators with respect to their phase-noise and power consumption.

1. INTRODUCTION

Nowadays, scientific contributions are either technology or topology oriented, the former looking rather for solutions into the silicon [1] and the latter looking for the solutions into the structure and the functionality. Seeking for inductors on chip with a quality factor larger than 10, or for transistors with a transition frequency over 100GHz, is of same importance as seeking for the concepts of how to place those high performance integrated components on the very same silicon. Accordingly, this paper is a contribution into the direction of topology driven research, eventually leading to a pure engineering approach in both analysis and design of high-performance bipolar voltage-controlled oscillators (VCO's).

As the voltage-controlled oscillators are playing a key role in analog front-ends with respect to their influence on the overall performance of fully integrated transceivers, any improvement in their design is highly appreciated.

Due to a limited voltage swing across the resonator and therefore limited phase-noise performance as well as the direct contribution of the base current shot-noise, designers are often hesitant to fully turn to bipolar technology and give up, almost native in the design of fully integrated VCO's, CMOS technology.

However, if it is possible to overcome the addressed disadvantages of using bipolar transistors for the active part of oscillators [2], and on the other hand to use their inherent advantages over MOS transistors, as a higher transconductance for the same operating point, a bipolar VCO could lead to a high-performance design.

The technique of quasi-capacitive tapping of LC oscillators, proposed in this paper, results in an increased voltage swing over the tank, while at the same time the transistor remains far from heavy saturation. Moreover, the base current shot noise and possible noise of the biasing network are reduced with the square of the quasi-tapping factor.

Unlike the technique of "real" tapping [3] of the LC-tank, where the tank is changed, "seen" by the active part of the oscillator, the technique of quasi-capacitive tapping corresponds only to a change in the portion of the effective transconductance of the oscillator's active part that is "seen" by the tank, and not to a change in the LC-tank itself.

This paper is divided into five sections. A well-known model of LC oscillators is briefly reviewed in the following section. The same model applied to the quasi-tapped LC oscillator is presented in Section 3, while the qualitative comparison of those two models, with respect to their power consumption and phase-noise, is the subject of Section 4. The last section summarizes the conclusions resulting from the presented analysis of the bipolar quasi-tapped VCO's.

2. NON-TAPPED LC-OSCILLATOR MODEL

Referring to the phase-noise of the oscillator as the key parameter characterizing its performance, it is necessary to perform step-by-step noise analysis of the complete oscillating system. Therefore, as a first step, the model of the oscillator, together with its tank, is introduced. This model is supposed to facilitate the subsequent noise analysis, finally resulting in a phase-noise model for bipolar voltage-controlled oscillators.

2.1 Oscillator model

In the analysis of the voltage-controlled oscillators under consideration, we will use a simple model, consisting of an active part, in the form of a transconductance amplifier implemented by a differential pair of an equivalent transconductance G_M, and an LC-tank, consisting of an inductance L, a capacitance C and their parasitic resistances, R_L, R_C and R_P, respectively, as shown in Fig. 1a.

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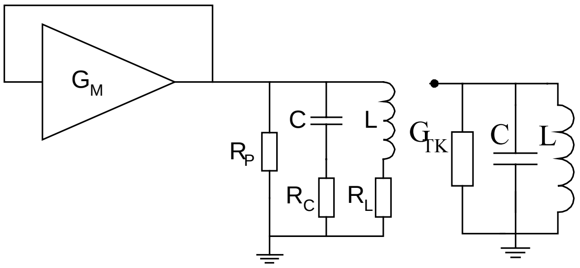

Fig. 1 (a) Simple LC-oscillator model. (b) LC-tank.

To simplify the analysis, the tank model of Fig. 1a is first transformed into the one of Fig. 1b. This is done by transforming the series connection of inductor, capacitor and their parasitics into a parallel one where the elements of Figs. 1a and 1b are related as:

$$G_{TK} = 1/R_P + R_L/(\omega_0 L)^2 + R_C(\omega_0 C)^2 \qquad (1)$$

2.2 Noise analysis

There are basically two main contributors to the output noise of the oscillator, being the resonator tank itself and the active part.

The contribution of the tank into the equivalent noise voltage spectral density can be calculated with the aid of the equivalent noise current spectral density of the lossy LC-tank, $\overline{I}^2_{N,TK}$, and the equivalent impedance of the ideal LC-tank, $Z(\Delta\omega)$.

With the assumption that $\Delta\omega << \omega_0$, the equivalent impedance of the ideal LC-tank, at an angular frequency $\omega_0+\Delta\omega$ that slightly deviates from the resonant angular frequency $\omega_0=2\pi f_0$, is:

$$Z(\omega_0+\Delta\omega) \cong \frac{-j\omega_0 L}{2\Delta\omega/\omega_0} \qquad (2)$$

Now, taking into account the contribution of the LC-tank as:

$$\overline{V}^2_{N,TK} = \overline{I}^2_{N,TK}\left|Z(\Delta\omega)\right|^2 = KT\frac{G_{TK}}{(\omega_0 C)^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2 \qquad (3)$$

and the contribution of the active part as:

$$\overline{V}^2_{N,A} = KT\frac{G_{TK}}{(\omega_0 C)^2}A\left(\frac{\omega_0}{\Delta\omega}\right)^2 \qquad (4)$$

the total voltage noise spectral density is calculated to be [4]:

$$\overline{V}^2_{N,TOT} = \overline{V}^2_{N,TK} + \overline{V}^2_{N,A}$$

$$\overline{V}^2_{N,TOT} = KT\frac{G_{TK}}{(\omega_0 C)^2}(1+A)\left(\frac{\omega_0}{\Delta\omega}\right)^2 \qquad (5)$$

where A accounts for both the additional noise of the active part and the excess conductance necessary for the safety start-up of the oscillations. Yet, K is Boltzmann's constant and T the absolute temperature.

3. QUASI-CAPACITIVE TAPPING OF LC-OSCILLATORS

The original contribution of this paper is the introduction of the quasi-tapped LC oscillator shown in Fig. 2. Here, the quasi-tapping capacitances C_A and C_B, serve to define the quasi-tapping ratio and not, as presented in some references [5], to facilitate biasing of the transistors in the active part of the oscillator. Unlike such a constellation, where the influence of these capacitors on both the LC-tank and the performances of the oscillator is fully neglected, in the upcoming analysis it will be shown that the role of the quasi-tapping capacitances C_A and C_B, in the oscillator under consideration, is substantially different, and that they determine the performances of the oscillator, being phase-noise and power-consumption, *equally* with the other elements in the circuitry.

Also, note that in this paper introduced term of quasi-capacitive tapping corresponds to a three-port oscillating system (Fig. 3), whereas in case of a non-tapped oscillator (Fig. 1) or a tapped one [3], the LC-tank – active part system is considered to be a two-port.

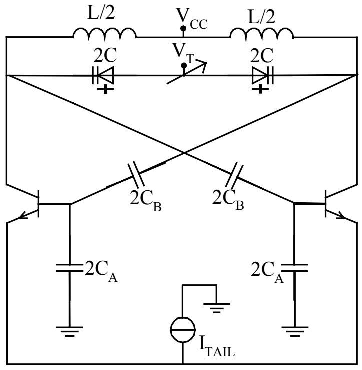

Fig. 2 Quasi-tapped LC-oscillator (concept; biasing not show)

In order to perform qualitative comparison between the simple LC oscillator, whose model is given in previous section, and the quasi-tapped LC oscillator, it is necessary to come up with the analytical expressions modeling the noisy nature of the latter as well. The simplified model of the oscillator is shown in Fig. 3.

The relation among the parameters of the oscillator can be summarized as:

$$n = 1+\frac{C_A}{C_B}, \quad G_M = g_m/2, \quad G_{M,TK} = G_M/n \qquad (6)$$

$$L_{TOT} = L, \quad C_{TOT} = C + \frac{C_A C_B}{C_A + C_B}, \quad \omega_0 = \frac{1}{\sqrt{L_{TOT}C_{TOT}}} \qquad (7)$$

where C_A and C_B are the quasi-tapping capacitances, n is the quasi-tapping factor, G_M the transconductance of the active part of the oscillator, $G_{M,TK}$ the transconductance seen by the LC-tank and g_m the transconductance of the bipolar transistors.

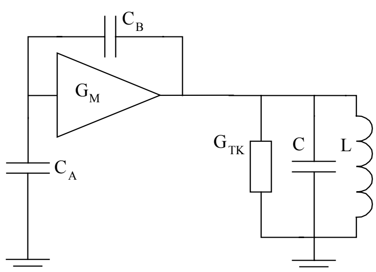

Fig. 3 Simplified model of a quasi-tapped oscillator.

3.1 Noise analysis

In the analysis to come, it is assumed that the oscillator operates in a near-linear fashion, such that the original noise close to the carrier contributes to a great extent to the total oscillator noise, compared to the other contributors, such as base-band noise and the one obtained after mixing from the other harmonics. As used for a *qualitative inspection* of the oscillators' phenomena, the introduced assumptions [4] enable easier interpretation of a rather complex noise generating mechanism in the VCO's.

Now, let us denote the main noise sources of a bipolar transistor as shown in Fig. 4a.

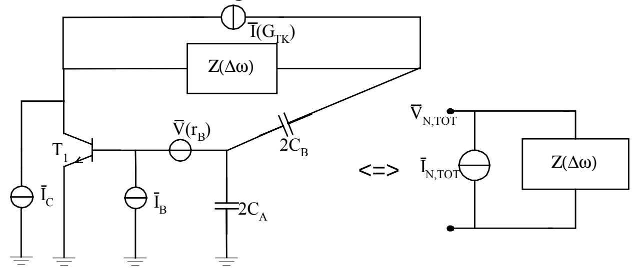

Fig. 4 (a) Noisy oscillator model. (b) Noisy tank model.

In order to switch to the equivalent model of Fig. 4b, it is necessary to transform the indicated noise sources to the corresponding LC-tank, ***Z(Δω).*** For the sake of brevity, and because of the apparent symmetry, only one half of the oscillator is depicted. However, in the calculations to come, the oscillator as a whole is being analyzed.

The equivalent noise current spectral densities at the output of the oscillator from the above noise sources, at the angular frequency $\omega_0+\Delta\omega$, are found as follows:

$$\bar{I}^2(r_B) = 2\bar{V}^2(r_B)n^2G_{TK}^2, \qquad \bar{V}^2(r_B) = 4KTr_B \tag{8}$$

$$\bar{I}^2(I_B) = \bar{I}_B^2/2n^2, \qquad \bar{I}_B^2 = 2qI_B \tag{9}$$

$$\bar{I}^2(I_C) = 1/2\bar{I}_C^2, \qquad \bar{I}_C^2 = 2qI_C \tag{10}$$

$$\bar{I}^2(G_{TK}) = 4KTG_{TK} \tag{11}$$

where $\bar{I}_B$ is the base current shot-noise, $\bar{I}_C$ the collector current shot-noise and $\bar{V}(r_B)$ the base resistance (r_B) thermal noise. The corresponding collector and base currents of the transistors are denoted to as I_C and I_B.

As considered to be uncorrelated, all noise sources add to the equivalent one as given by Eq. (12) and Fig. 4b.

$$\bar{I}_{N,TOT}^2 = \bar{I}^2(r_B) + \bar{I}^2(I_B) + \bar{I}^2(I_C) + \bar{I}^2(G_{TK}) \tag{12}$$

Accordingly, the output voltage noise spectral density is:

$$\bar{V}_{N,TOT}^2 = \bar{I}_{N,TOT}^2|Z(\Delta\omega)|^2 = KT\frac{G_{TK}}{(\omega_0 C_{TOT})^2}\left(\frac{\omega_0}{\Delta\omega}\right)^2[1 + \frac{qI_C}{4KTG_{TK}} + 2n^2r_BG_{TK} + \frac{qI_B}{n^2 4KTG_{TK}}] \tag{13}$$

$$\bar{V}_{N,TOT}^2 = KT\frac{G_{TK}}{(\omega_0 C_{TOT})^2}(1+A_T)\left(\frac{\omega_0}{\Delta\omega}\right)^2 \tag{14}$$

$$A_T = 2n^2r_BG_{TK} + g_m(1+1/\beta)/4G_{TK}, \qquad \beta = I_C/I_B \tag{15}$$

From the start-up condition Eq. (6) and assuming $\beta >> 1$, factor A_T – the noise factor of the active part – can be re-written as:

$$A_{T,S-UP} = n(1/2 + r_Bg_{ms-up}) \tag{16}$$

or for the safety start-up, corresponding to the case with the excess loop-gain larger than one – in this case k – it is given as:

$$A_{T,S\_S-UP} = n(k/2 + r_Bg_{ms\_s-up}/k) = n(k/2 + r_Bg_{ms-up}) \tag{17}$$

Note, that indexes *s-up* and *s_s-up* correspond to the start-up and the safety start-up conditions of the oscillator.

4. QUASI-TAPPED VS. NON-TAPPED OSCILLATORS

Let us denote $\mathcal{L}$, V_S, and $\bar{V}_{TOT}$ as phase noise, signal amplitude and overall output noise of the oscillator, where index $_{QT}$ will be used for quasi-tapped and index $_{NT}$ for non-tapped oscillators. Defined as the ratio of the power in a 1Hz bandwidth at a frequency $f_0+\Delta f$ and the carrier power, the phase-noise is given as:

$$\mathcal{L} = \frac{\bar{V}_{N,TOT}^2}{V_S^2/2} \tag{18}$$

In order to perform full and fair comparison between the two oscillator models, we will refer to the cases such as the start-up condition, the maximum phase-noise or maximum power condition, and the same power condition. For this purpose, it is necessary to transform the expression for the phase-noise in a form suitable for straightforward comparison of performances of the given oscillator prototypes. The LC-tanks of the quasi-tapped and non-tapped oscillator models are assumed to be identical.

Combining Eqs. (14) and (18), the phase-noise of a quasi-tapped and a non-tapped (n=1) oscillator appears to be:

$$\mathcal{L}_{QT} \propto \frac{1 + 2n^2r_BG_{TK} + g_{m,QT}/4G_{TK}}{n^2V_{S,NT}^2} \tag{19}$$

$$\mathcal{L}_{NT} \propto \frac{1 + 2r_BG_{TK} + g_{m,NT}/4G_{TK}}{V_{S,NT}^2} \tag{20}$$

With the aid of Eq. (6) and for an arbitrary distance from the start-up condition, it can be written as:

$$\mathcal{L}_{QT} \propto \frac{1 + n\cdot(k/2 + r_Bg_{ms-up,QT})}{n^2k^2V_{S,S-UP,NT}^2} \tag{21}$$

$$\mathcal{L}_{NT} \propto \frac{1 + k/2 + r_Bg_{ms-up,NT}}{k^2V_{S,S-UP,NT}^2} \tag{22}$$

where parameter $k=G_{M,TK}/G_{TK}$ defines how far the oscillator is from the start-up condition. As this is a key parameter used in the following analysis, it is worth explaining, in more detail, its meaning and importance.

In its simplest form, k is the loop-gain of the oscillator seen as a positive feedback amplifier. Also, it is the excess of the negative conductance, necessary for the compensation of the losses in the LC-tank. Namely, if the tank conductance is G_{TK}, then for the start-up of the oscillations the negative conductance of the active part must be $G_{M,TK}=kG_{TK}$, where k is larger then one, usually for a safety start-up set to a value of two.

Letting A_{NT} be

$$A_{NT}(k) = k/2 + r_Bg_{ms-up,NT} \tag{23}$$

the expressions for the phase-noise are transformed into:

$$\mathcal{L}_{QT}(k) \propto \frac{1 - (n-1)\cdot n\cdot k/2 + A_{NT}(k)n^2}{n^2k^2} \tag{24}$$

$$\mathcal{L}_{NT}(k) \propto \frac{1 + A_{NT}(k)}{k^2} \tag{25}$$

Now, the comparison appears to be quite simple. Namely, the ratio between phase-noise of a non-tapped and a quasi-tapped oscillator $R(k_1,k_2)$, can straightforwardly be defined as:

$$R(k_1,k_2) = \mathcal{L}_{NT}(k_2)/\mathcal{L}_{QT}(k_1) \tag{26}$$

$$R(k_1,k_2) = \frac{2n^2k_1^2}{2 - (n-1)\cdot n\cdot k_1 + 2n^2A_{NT}(k_1)}\frac{1 + A_{NT}(k_2)}{k_2^2} \tag{27}$$

where index $_1$ corresponds to a quasi-tapped and index $_2$ to a non-tapped oscillator model.

The validity of the expression is supported letting $n=1$ and $k_1=k_2$, when the resulting difference is 0dB, i.e., there is no difference between two identical non-tapped oscillators.

The operating conditions that will be used in the analysis are the one for the same power consumption ($k_1=nk_2$), and the one for the same distance from the start-up condition ($k_1=k_2=k$), enabling us to examine how the phase-noise of the oscillator is changed in all the relevant conditions under which it might operate. Without loss of generality, and for easier interpretation of the upcoming analysis, we will assume that the quasi-tapping factor equals two, i.e., $n=2$.

Now, the phase-noise ratio for the same power consumption is given as:

$$R(k_1,2k_1) = \frac{1+A_{NT}(2k_1)}{1-k_1+4A_{NT}(k_1)} = \frac{1+k_1+c}{1+k_1+4c} < 1 \qquad (28)$$

where c is a positive constant defined as $c=r_B\, g_{ms\text{-}up,NT}$.

As expected, *a non-tapped oscillator has always better performance than a quasi-tapped oscillator, with respect to the phase-noise for the same power consumption*. For example, if $k_1=1$ – the start-up condition – r_B *is* 26Ω and $g_{ms\text{-}up,NT}$ is 3.8*mS*, there is a difference in phase-noise of 0.7dB in favor of the non-tapped oscillator.

The given analysis can be graphically supported by means of, for the first time introduced, k-rails diagram, shown in Fig. 5. If the arrows in the figure are regarded as lines of constant loop-gain, phase-noise and power consumption, than for the same power consumption $P_1=P_2$ and accordingly $k_2=nk_1$ (point A and point B), holds $PN_2>PN_1$, i.e. phase-noise ($\mathcal{L}=1/PN$) of a non-tapped oscillator is better then the one of a quasi-tapped, as already indicated by Eq. (28). Note that the left rail corresponds to the non-tapped and the right rail to the quasi-tapped oscillator.

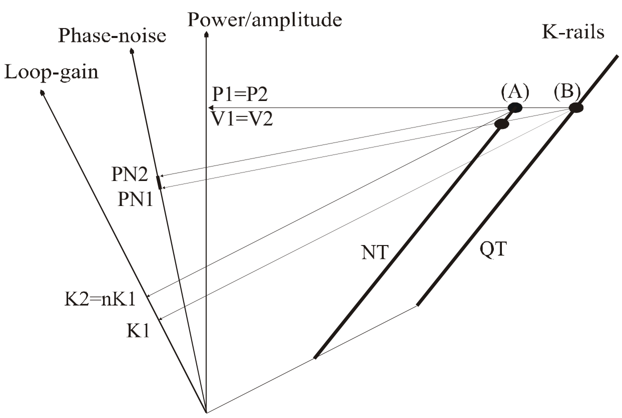

Fig. 5 K-rails diagram - the same power consumption.

In a similar manner, a phase-noise ratio, for the same excess negative conductance and $n=2$, is given as:

$$R(k,k) = \frac{4+4A_{NT}(k)}{1-k+4A_{NT}(k)} = \frac{4+2k+4c}{1+k+4c} > 1 \qquad (29)$$

which is obviously larger than 0*dB*, proving that *a quasi-tapped oscillator has always better performance than a non-tapped oscillator, with respect to the phase-noise for the same loop-gain.* Using the same example as in the previous case, there is a difference in phase-noise, for $k=k_{MAX}=4$, of 3.6dB in favor of a quasi-tapped oscillator. This is shown in Fig. 6, where for the phase-noise corresponding to the points A and B ($k_2=k_1=k$), holds $PN_2<PN_1$. Also, the region of a superior quasi-tapped VCO performance (B – C) is easily recognized in the diagram.

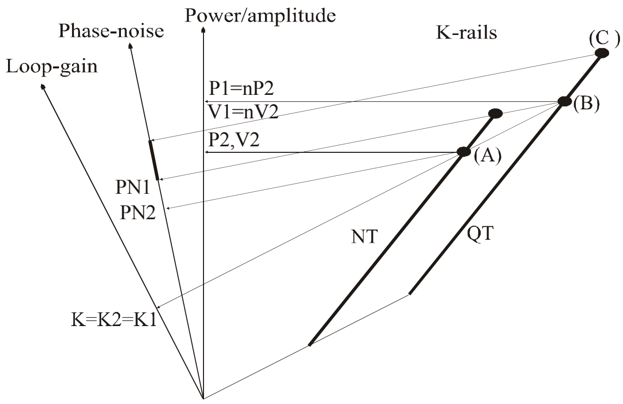

Fig. 6 K-rails diagram - the same loop gain.

Finally, note that both the exposed concept and the obtained results are fully confirmed by the CADENCE simulation tool SpectreRF.

5. CONCLUSIONS

The analytical expressions, presented in this paper, serve to give an insight into the trade-offs between quasi-tapped and non-tapped oscillators with respect to their phase-noise and power consumption.

It is shown that non-tapped oscillators are "the best" low power solution, as they have better phase-noise performance than quasi-tapped oscillators for the same power consumption.

Also, it is shown that the quasi-tapped oscillators are "the best" maximum performance solution, as they can achieve higher phase-noise than non-tapped oscillators for a certain increase in power. In the case of non-tapped oscillators, the increase in power doesn't help due to a limited voltage swing. Applying quasi-capacitive tapping between the active part of the oscillator and the LC-tank, the voltage swing over the tank can be increased accordingly, while at the same time the transistor remains far from heavy saturation

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