Dynamic Translinear Circuits — An Overview

Jan Mulder Wouter A. Serdijn Albert C. van der Woerd
Arthur H.M. van Roermund

Abstract

Dynamic translinear circuits and log-domain filters form a promising and challenging approach to meet the dynamic range limitations that conventional analogue implementation techniques are facing due to ever lower supply voltages, low power consumption and high-frequency demands. This paper aims to give an overview of this young, yet rapidly developing, circuit paradigm. Emphasis is placed on methods for analysis and synthesis and on state-of-the-art results obtained for both linear and non-linear applications.

1 Introduction

Due to the ongoing trends to lower supply voltages and low power operation, the area of *analogue integrated filters* is facing serious challenges [?, ?]. The maximal dynamic range achievable using conventional filter implementation techniques, such as opamp-MOSFET-C, transconductance-C and switched-capacitor, becomes severely restricted by the supply voltage. In ultra-low-power environments, linear resistors become too large for on-chip integration. Finally, the situation is complicated by high-frequency demands and the fact that the filter transfer function has to be tuneable to compensate for process tolerances.

In the area of *continuous-time* filters, a promising approach to meet these challenges is provided by the class of 'Translinear Filters'. Due to the encouraging expectations, research efforts have increased rapidly and TransLinear (TL) filter design has become a trend. This is illustrated in Fig. 1. In a general context, translinear filters form a sub-class of an encompassing class of companding networks that exhibit a theoretically linear frequency-dependent transfer function externally even though the internal signal path contains non-linear elements. An excellent general overview of companding filters can be found in [?].

Translinear filters were originally introduced by Adams in 1979 [?]. Since Adams at the time did not recognise the TL nature of these circuits, he coined

[0]The authors are with the Electronics Research Laboratory, Department of Electrical Engineering, Delft University of Technology, Delft, The Netherlands.

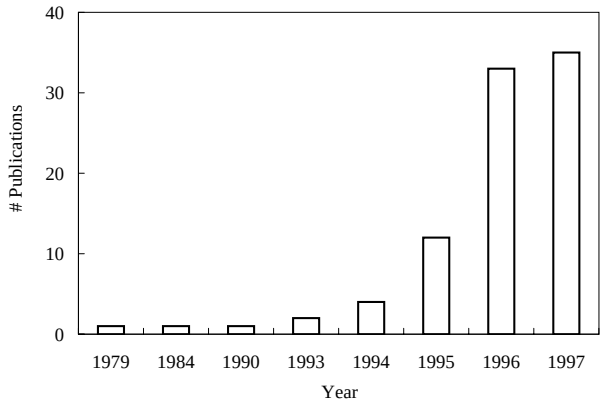

Figure 1: Publications on dynamic translinear circuits.

the term 'log-Domain Filters', based on the logarithmic relation between the voltages and currents. For many years, the idea of log-domain filtering was to gather dust. In 1990, Seevinck independently reinvented the TL filter concept, which he denoted by the term 'Current-mode Companding' [**?**]. The filters presented by both Adams and Seevinck were first-order. The expansive interest in TL filters really took off in 1993, when Frey published a synthesis method enabling the design of higher-order log-domain filters [**?**]. In addition, Frey proposed a more general class of TL filters, which he termed 'Exponential State-Space Filters' [**?**].

From that time, many other researchers began to investigate these filters. Toumazou *et al.* published an implementation in weak inversion MOS, showing the potential for low-power operation [**?**]. The first experimental results were published by Perry and Roberts [**?**]. In addition, they proposed an alternative synthesis method based on the simulation of *LC* ladder filters. The first experimental results in subthreshold MOS were presented by Ngarmnil *et al.* [**?**]. Punzenberger *et al.* demonstrated the suitability for low-voltage applications [**?**] and the favourable dynamic range specification resulting from class AB operation: 65 dB at 1.2 V supply voltage [**?**]. Different synthesis methods were proposed by various researchers [**?**, **?**, **?**, **?**, **?**, **?**]. A general analysis method was published by Mulder *et al.* [**?**], who also coined the term 'Translinear Filter'. Alternative analysis methods were described in [?, **?**]. Application of the underlying design principle to non-linear dynamic functions was proposed by various researchers. These applications include oscillators [**?**, **?**, **?**], RMS-DC converters [**?**, **?**], mixer-filter combinations [**?**, **?**] and phase-locked loops [**?**, **?**, **?**]. A generalisation to strong inversion MOS was proposed independently by Mulder *et al.* [**?**] and Payne *et al.* [**?**]. At present, many research efforts are developed in the area of noise analysis [?, **?**, **?**, ?, **?**, **?**, **?**], and other second-order effects [**?**].

This paper aims to give an overview of the complete field of dynamic translinear circuits. The emphasis is on structured design methods and principles,

rather than on specific circuit implementations. The static and dynamic TL principles are reviewed in Section 2. Section 3 gives an overview of analysis methods. The general class of TL filters contains several different types. In Section 4, the correspondences and differences between log-domain, tanh, and sinh filters are treated. The theoretical relevance, or better, irrelevance, of a generalisation of the DTL principle to strong inversion MOS is discussed in Section 5. Section 6 presents an overview on synthesis methods. Finally, an overview of state-of-the-art results is presented in Section 7.

2 Translinear principles

Translinear circuits can be divided into two major groups: static translinear (STL) and dynamic translinear (DTL) circuits. Static TL circuits realise static transfer functions, both linear and non-linear; DTL circuits realise frequency-dependent (transfer) functions, i.e., differential equations (DEs). The underlying principles of STL and DTL circuits are reviewed in this section.

2.1 Static translinear principle

Translinear circuits are based on the exponential relation between voltage and current, characteristic for the bipolar transistor and the MOS transistor in the weak inversion region . The collector current I_C of a bipolar transistor in the active region is given by:

$$I_C = I_s \mathrm{e}^{V_{BE}/U_T}, \tag{1}$$

where all symbols have their usual meaning.

The TL principle applies to loops of semiconductor junctions. A TL loop is characterised by an even number of junctions [?, ?]. The number of devices with a clockwise orientation equals the number of counter-clockwise oriented devices. An example of a four-transistor TL loop is shown in Fig. 2. It is assumed that the transistors are somehow biased at the collector currents I_1 through I_4. When all devices operate at the same temperature, this yields the familiar representation of TL loops in terms of products of currents:

$$I_1 I_3 = I_2 I_4. \tag{2}$$

This generic TL loop equation is the basis for a wide variety of static electronic functions, which are theoretically temperature- and process-independent.

2.2 Dynamic translinear principle

The introduction of the capacitance as a basic element of TL networks significantly extends the applicability of these circuits. As a result, DEs can be realised; both linear DEs, describing linear filters, and non-linear DEs, e.g., oscillators and PLLs. The term 'Dynamic Translinear' was coined in [?] to describe this class of circuits. This expression emphasises the TL nature of these

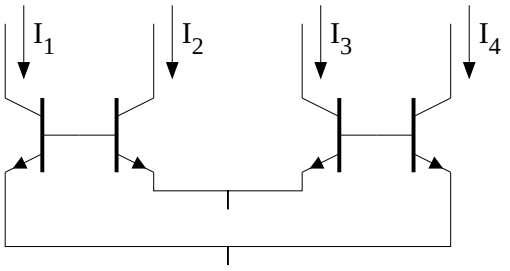

Figure 2: A four-transistor translinear loop.

circuits, which has proven to be a distinct advantage with respect to structured analysis and synthesis.

The DTL principle can be explained with reference to the sub-circuit shown in Fig. 3. Using the current-mode approach, this circuit is described in terms of the collector current I_C and the capacitance I_{cap} flowing through the capacitance C. Note that the dc voltage source V_{const} does not affect I_{cap}. An expression for I_{cap} can be derived from the time derivative of eqn (1) [?, ?]:

$$I_{cap} = CU_T \frac{\dot{I}_C}{I_C}, \tag{3}$$

where the dot represents differentiation with respect to time.

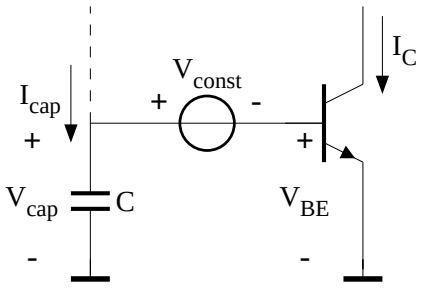

Figure 3: Principle of dynamic translinear circuits.

Equation (3) shows that I_{cap} is a non-linear function of I_C and its time derivative $\dot{I}_C$. A better understanding of eqn (3) is obtained by slightly rewriting it:

$$CU_T \dot{I}_C = I_{cap} I_C. \tag{4}$$

Equation (4) directly states the DTL principle: *A time derivative of a current can be mapped onto a product of currents.* At this point, the conventional (static) TL principle comes into play, for, the product of currents on the right-hand side (RHS) of eqn (4) can be realised very elegantly by means of the STL

principle. Thus, the implementation of (part of) a DE becomes equivalent to the implementation of a product of currents.

3 Analysis

In almost all publications on DTL circuits, the emphasis has been on synthesis. Both structured design methods and new circuit realisations have been presented. Nonetheless, although synthesis is more powerful than analysis, it must go together with a generally applicable analysis method in the same domain. Only when this condition is met, the full potentials of a synthesis method can be exploited.

In this section, an overview is given of the analysis methods proposed in literature. The methods are treated in the chronological order in which they have appeared in literature.

3.1 Voltage-mode analysis

In [?], Adams not only presented a synthesis method, but also proposed an analysis method. The first step of the analysis procedure is to write down the node equations from the large-signal ac model of the filter. Next, the equations containing the derivative of a capacitance voltage are multiplied by an exponential term. Using the chain rule, the isolated derivatives can be eliminated, as follows:

$$\dot{V}_{cap} \quad \longrightarrow \text{ multiply by } \mathrm{e}^{\frac{V_{cap}}{U_T}} \longrightarrow \quad \dot{V}_{cap}\mathrm{e}^{\frac{V_{cap}}{U_T}} = U_T \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{\frac{V_{cap}}{U_T}}. \qquad (5)$$

The intermediate voltages have to be eliminated, such that a single equation results, expressing the relation between the compressed input and output voltage. Unfortunately, Adams does not give a systematic method to accomplish this [?].

Finally, application of a logarithmic transformation yields the DE describing the circuit in question. Adams proposed his analysis procedure in the context of log-domain filters. However, in principle, the voltage-mode analysis method can be applied to the analysis of tanh and sinh filters, as well. In that case, the required transformations are the inverse hyperbolic tangent and sine function, respectively.

Implicitly, Adams' method has been applied in numerous publications to verify parts of transistor level implementations. In [?], an example can be found of the voltage-mode analysis of a complete second-order TL filter. Note that in most papers, the analysis process is simplified by direct substitution of the overall V-I transfer function of prevalent building blocks, such as the E^+, E^-, T, S and $S2$ blocks introduced in [?]. This reduces the number of intermediate voltages and node equations.

3.2 Small-signal analysis

A very simple way to calculate the transfer function of a complete filter is to analyse the small-signal equivalent circuit, see, e.g., [?]. Since, by definition, a small-signal analysis results in a linear transfer function, this method yields the correct DE only when the DTL circuit under consideration is externally linear and properly designed. The large-signal linearity cannot be proven and has to be verified in another way. Numerical simulations can provide some insight. Obviously, small-signal analysis cannot be applied to non-linear DTL circuits.

3.3 Global translinear analysis

A large-signal analysis method was presented by the authors in [?]. This *current-mode* method is based on a TL approach and is believed to be completely general. It has been tested with success on all published log-domain, tanh and sinh filters. Basically, the only difference between STL and DTL circuits is the presence of capacitances. The capacitance *currents* form the key to the analysis of DTL circuits.

Static TL circuits can be analysed through the method described in [?]. The first step is to express all collector currents in terms of the current sources, which are connected to the nodes of the TL core. The collector currents are linear combinations of the input, dc bias and output currents, and of some intermediate currents in case of multiple-loop circuits. Once the collector currents are found, the TL loop equations are derived. These are given by equations like (2). The last analysis step is to solve the system of TL loop equations for the output current(s) by eliminating the intermediate currents.

In DTL circuits, some capacitors are connected to the nodes of the TL core. Consequently, the node currents are determined as well by the currents flowing through these capacitors. Hence, the capacitance currents appear in the TL loop equations. From this point of view, the capacitors can be regarded as being a special kind of current source.

To solve the system of loop equations, the capacitance currents have to be eliminated. To this end, expressions for the capacitance currents have to be found. Fortunately, this is simple. A capacitance connected to a node of the TL core will always form a loop with one or more base-emitter junction in series. This is illustrated in Fig. 4. The capacitance voltage V_{cap} can be expressed in terms of the base-emitter voltages, which in turn are expressed in terms of the collector currents flowing through these transistors. The capacitance current I_{cap} can now be calculated from the constitutive law by taking the derivative of V_{cap} with respect to time. Thus, a very simple *current-mode* equation is obtained:

$$I_{cap} = CU_T \sum_i \pm \frac{\dot{I}_{C,i}}{I_{C,i}}. \tag{6}$$

The $\pm$ sign of each term depends on the orientation of the corresponding transistor. To analyse a DTL circuit, eqn (6) has to be applied to each capacitance

in the circuit to find an expression for the current flowing through it. Finally, elimination of the intermediate currents yields the DE describing the output current.

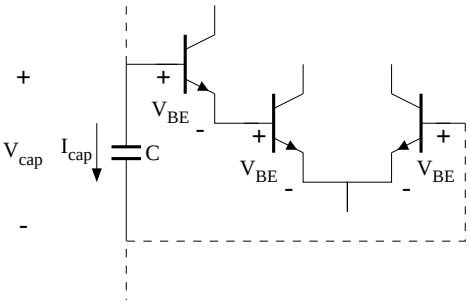

Figure 4: A capacitance in (a part of) a translinear loop.

An example of the application of the global translinear analysis method can be found in [?], where a second-order TL low-pass filter, designed by Frey [?], is analysed.

3.4 Analysis based on Bernoulli's DE

An alternative *current-mode* analysis method has been proposed by Drakakis *et al.* in [?]. This method can be used to analyse log-domain filters based on the generic structure shown in Fig. 5. The currents I_{u_k}, where $k \in [1, \ldots, n]$, and n denotes the order of the filter, determine the transfer function of the filter. The currents I_{o_k} are dc bias currents.

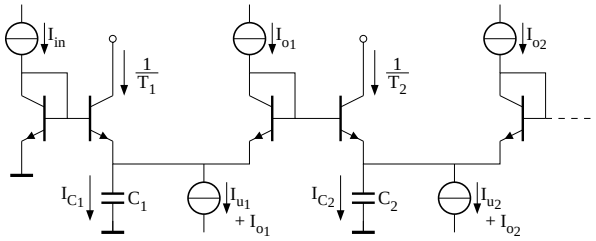

Figure 5: Generic structure that can be analysed with the analysis method based on Bernoulli's differential equation.

The analysis procedure is based on the 'Bernoulli cell', shown in Fig. 6, which is the basic element of the generic structure shown in Fig. 5. In general, the Bernoulli cells are described by a first-order DE:

$$C_k U_T \frac{\mathrm{d}}{\mathrm{d}t} \ln c_k I_{in} T_1 \ldots T_k + I_{u_k} = \frac{1}{T_k}, \tag{7}$$

where I_{in} is the input current, c_k is a constant with dimension $[\mathrm{A}^{k-1}]$, C_k is a capacitance and $1/T_k$ a collector current, as shown in Fig. 6. Definition of a current $I_{w_k} = c'_k I_{in} T_1 \ldots T_k$, where c'_k is a constant with dimension $[\mathrm{A}^k]$, and substitution in eqn (7) yields:

$$C_k U_T \dot{I}_{w_k} + I_{u_k} I_{w_k} = \frac{c'_k}{c'_{k-1}} I_{w_{k-1}}. \tag{8}$$

By definition, I_{w_0} equals I_{in}.

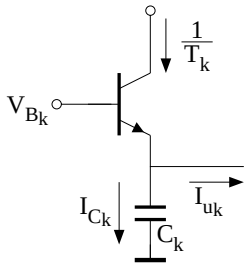

Figure 6: Bernoulli cell.

To analyse a log-domain filter, first, the currents I_{u_k} have to be determined using STL analysis techniques. Next, application of eqn (8) to all Bernoulli cells yields a set of n first-order DEs; a state-space description of the DTL circuit in question.

3.5 State-space translinear analysis

State-space descriptions can be used to break down a high-order DE into a system of first-order DEs. In TL filters, the state-space approach can be applied beneficially for analysis purposes. Using the state-space translinear analysis method described in this section [?], linear equations are obtained at an earlier stage of the analysis. This limits the significant intermediate expression swell, which is an inherent characteristic of the global translinear analysis method.

In general, it is necessary to choose state variables in order to find a state-space description. For TL filters, the capacitance *voltages* are inconvenient state variables. Since TL filters are *current-mode* circuits, a better choice is to use the *currents* obtained from an exponential V-I expansion of the capacitance voltages, applying the exponential law describing the bipolar transistor.

As an example, consider the second-order filter shown in Fig. 7. In this filter, expansion of the voltage V_{C_1} across capacitance C_1 is already implemented by means of the output transistor Q_6. Therefore, the collector current I_{out} of Q_6 is chosen as the first state variable. Note that Q_5 merely acts as a dc voltage source.

The voltage V_{C_2} across the second capacitance C_2 is not expanded within the filter, but this can be accomplished by adding a *fictitious* sensing transistor

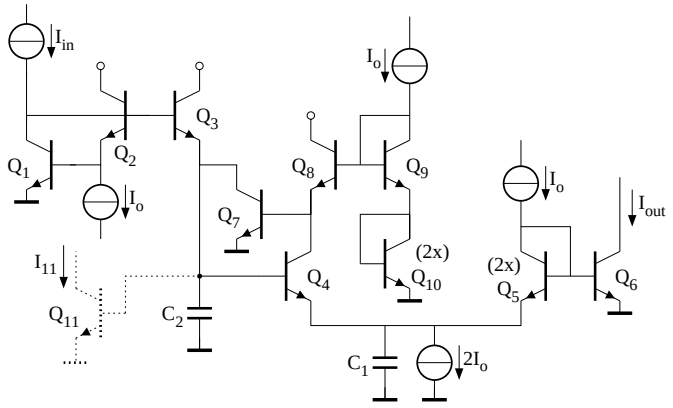

Figure 7: Sensing the states of a translinear filter.

Q_{11}, as shown in Fig. 7. The collector current I_{11} of this transistor is the second state variable to be used.

The actual filter shown in Fig. 7 consists of two disjunct TL loops: Q_1 through Q_6 and Q_7 through Q_{10}. By adding Q_{11}, the first loop, Q_1–Q_6, is broken into two coupled second-order loops, i.e., Q_1-Q_2-Q_3-Q_{11} and Q_{11}-Q_4-Q_5-Q_6. Hence, the filter can be described by a set of three loop equations and two expressions for the capacitance currents I_{C_1} and I_{C_2}, given by:

$$I_{in} I_o = (I_7 + I_{C_2})\, I_{11}, \tag{9}$$

$$I_{11} I_o = 2\,(I_o + I_{C_1})\, I_{out}, \tag{10}$$

$$2I_7\,(I_o + I_{C_1}) = I_o^2, \tag{11}$$

$$I_{C_1} = CU_T \frac{\dot{I}_{out}}{I_{out}}, \tag{12}$$

$$I_{C_2} = CU_T \frac{\dot{I}_{11}}{I_{11}}, \tag{13}$$

where the intermediate current I_7 is the collector current of Q_7. The factors 2 in eqns (10) and (11) are due to the emitter area scaling of Q_5 and Q_{10}.

Next, we have to find expressions for $\dot{I}_{out}$ and $\dot{I}_{11}$ in terms of I_{in}, I_{out} and I_{11}. An equation for $\dot{I}_{out}$ is found by eliminating I_{C_1} from (10) using (12). This yields:

$$CU_T \dot{I}_{out} = I_o \left(\tfrac{1}{2} I_{11} - I_{out}\right). \tag{14}$$

To find an expression for $\dot{I}_{11}$, we first eliminate I_7 from (9) and (11). From the resulting equation, the capacitance currents I_{C_1} and I_{C_2} can be eliminated using (12) and (13), after the derivative $\dot{I}_{out}$ has been eliminated from (12) by using (14). This yields the second equation of the state-space description:

$$CU_T \dot{I}_{11} = I_o\,(I_{in} - I_{out}). \tag{15}$$

Thus, a complete current-mode state-space description of the TL filter shown in Fig. 7 is given by eqns (14) and (15). The overall transfer function can be obtained easily using well-known techniques.

For the state-space analysis of tanh and sinh filters, instead of the common-emitter (CE) output stage, the generic output stages of these filters, shown in Fig. 8(b,c), must be used to sense the filter states. The current I_{dc} is a dc bias current.

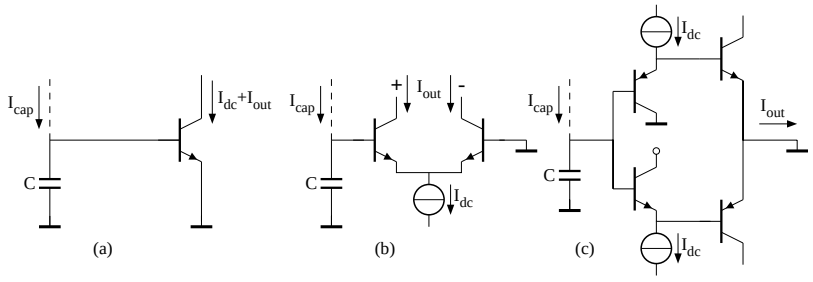

Figure 8: Generic output structures of (**a**) log-domain, (**b**) tanh, and (**c**) sinh filters.

4 Classes of translinear filters

In all TL filters, the voltages are logarithmically related to the currents. Therefore, these circuits are in some way instantaneous companding. Figure 9 shows the general block schematic of an instantaneous companding integrator [?]. In DTL circuits, the internal integrator is a linear capacitance. The expander E expands the output voltage of this integrator into a current, exploiting the exponential V-I transistor transfer function. Several types of TL filters can be distinguished within the general class of DTL circuits based on the particular implementation of E. Next to the most prevalent class of log-domain filters, the two classes of tanh and sinh filters have been proposed by Frey [?]. In this section, we describe their characteristics, which can be derived from the generic output structures, depicted in Fig. 8.

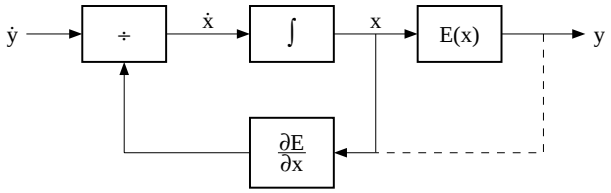

Figure 9: General block schematic of an instantaneous companding integrator.

4.1 log-domain filters

Most published DTL circuits are based on the CE output stage shown in Fig. 8(a), characteristic for the class of log-domain filters. The transfer function from the capacitance voltage V_{cap} to the output current I_{out} is given by the well-known exponential law (1). In other words, E equals $\exp x$. The companding characteristics of a TL filter can be derived from the second order derivative of E with respect to x, denoted by E''. Without loss of generality, $x = 0$ is considered to be the quiescent point of the integrator shown in Fig. 9. Applying a strict definition of companding, E'' should be strictly positive for $x > 0$ and strictly negative for $x < 0$. Figure 10 displays E'' for the output stages shown in Fig. 8. For log-domain filters, a comparison of $E'' = \exp x$ with the strict definition of companding reveals that these circuits are indeed companding for $x > 0$; however, for $x < 0$ the exponential function constitutes a compression instead of an expansion. For a symmetrical output current, the overall behaviour of the CE output stage implies a compression rather than an expansion of the peak-to-peak signal swings [**?**].

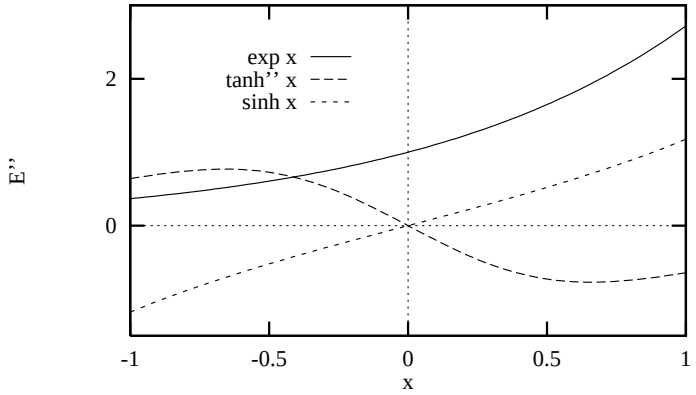

Figure 10: The second-order derivatives of the V-I transfer functions of the output stages shown in Fig. 8.

From a current-mode point of view, the most important characteristic of a DTL output structure is the current-mode expression for the capacitance current I_{cap}. For log-domain filters, I_{cap} is given by eqn (3), where $I_C = I_{dc} + I_{out}$. As shown in Section 2, a linear derivative $\dot{I}_{out}$ is obtained by multiplying I_{cap} by $I_{dc} + I_{out}$.

A favourable property of log-domain filters is that a linear damping term can be implemented by the connection of a dc current source I_o in parallel to a capacitance. This can be explained from eqn (4). If instead of I_{cap}, $I_{cap} + I_o$ is multiplied by $I_{dc} + I_{out}$, an additional term $I_o \cdot (I_{dc} + I_{out})$ is generated. The first term $I_o I_{dc}$ represents a dc offset current. The second term $I_o I_{out}$ results in a finite negative pole.

Typically, log-domain filters operate in class A. The actual ac signal I_{out} is

superposed on a dc bias current I_{dc}. As a consequence, the output signal swing is limited to $I_{out} > -I_{dc}$. This limitation is single sided,[1] which is advantageous if a-symmetrical input wave-forms have to be processed. This characteristic can be exploited to enable class AB operation [?, **?**]. Using a class AB set-up, see Fig. 11, the dynamic range can be enlarged without increasing the quiescent power consumption. Using a current splitter, the input current I_{in} is divided into two currents I_{in1} and I_{in2}, which are both strictly positive, and related to I_{in} by: $I_{in} = I_{in1} - I_{in2}$. The current splitter impresses a constant geometric or harmonic mean on I_{in1} and I_{in2}. Next, I_{in1} and I_{in2} can be processed by two class A log-domain filters. It is important to note that class AB operated log-domain filters do satisfy the strict definition of companding due to the fact that only positive currents are processed, i.e., x is never negative.

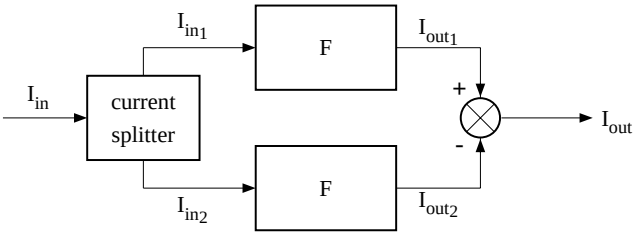

Figure 11: Set-up for class AB operation.

4.2 tanh filters

Instead of a single transistor in CE configuration, the class of tanh filters is characterised by a differential pair output structure [?], see Fig. 8(b). The name of this class of filters is derived from the well-known hyperbolic tangent V-I transfer function. The second-order derivative E'' is shown in Fig. 10 and demonstrates that tanh filters are not companding at all [**?**]. The differential pair implements a *compression* function.

The tail current of the differential pair is a dc current I_{dc}, and therefore, tanh filters also operate in class A. The output current I_{out} is the difference of the two collector currents. The output swing is limited to $-I_{dc} < I_{out} < I_{dc}$. Since this interval is symmetrical, the class AB set-up shown in Fig. 11 cannot be applied to tanh filters.

Using (6), the capacitance current I_{cap} is found to be:

$$I_{cap} = CU_T \left(\frac{\dot{I}_{out}}{I_{dc} + I_{out}} - \frac{-\dot{I}_{out}}{I_{dc} - I_{out}} \right). \tag{16}$$

A linear derivative $\dot{I}_{out}$ is obtained by multiplying this equation by $(I_{dc} +$

[1] It is important to note that limitations of the signal swings are not only determined by the type of output stage, but by the complete TL filter circuit.

$I_{out})(I_{dc} - I_{out})$:

$$2CU_T I_{dc}\dot{I}_{out} = I_{cap}(I_{dc} + I_{out})(I_{dc} - I_{out}). \tag{17}$$

Comparing eqns (4) and (17), we can see that the RHS of (17) is third-order, whereas the RHS of (4) is only second-order. Consequently, in general, TL loops of a higher order are required to implement a tanh filter, resulting in a more complex circuit. In addition, a linear loss cannot be implemented by a dc current source connected in parallel to a capacitance. This leads us to the conclusion that tanh filters do not seem to have any advantages over log-domain filters.

4.3 sinh filters

The third class of DTL circuits proposed in literature is formed by the sinh filters [?]. The output structure, shown in Fig. 8(c), is a complete second-order TL loop. It implements the geometric mean function $I_{dc}^2 = I_{out1}I_{out2}$. The actual output current I_{out} is the difference of I_{out1} and I_{out2}. Since both I_{out1} and I_{out2} are always positive, the sinh output structure operates in class AB, which is beneficial with respect to the dynamic range. The V-I transfer function of the output structure is a hyperbolic sine function. Figure 10 displays $E'' = \sinh x$ and shows that the sinh output stage implements a genuine expansion function.

The current-mode expression for the capacitance current I_{cap} is given by:

$$I_{cap} = CU_T \frac{\dot{I}_{out}}{I_{out1} + I_{out2}}. \tag{18}$$

A linear derivative $\dot{I}_{out}$ is obtained by multiplying I_{cap} by the sum $I_{out1} + I_{out2}$. It is interesting to note that the voltage V_{cap} and the current $I_{out1} + I_{out2}$ are related through a hyperbolic cosine function; the first-order derivative of E with respect to x.

5 Dynamic voltage-translinear circuits

A generalisation of the DTL principle to strong inversion MOS transistors was proposed independently by the authors [?] and Payne *et al.* [?]. These 'Dynamic Voltage-translinear[2]' (DVTL) circuits add the capacitance as a basic element to the conventional class of (static) voltage-translinear (VTL) circuits [?].

This section discusses the practical significance of DVTL circuits. Though of crucial importance, the square law conformance of MOS transistors in modern IC processes is not treated here; see, e.g., [?]. Instead, the practical relevance of possible classes of DVTL circuits is discussed. In particular, three classes of DVTL circuits are distinguished, which are based on the generic output stages

[2]The term 'Voltage-Translinear' proposed in [?] is used here as it clearly distinguishes between TL principles based on the exponential law and VTL principles based on the square law, as opposed to the term 'MOS Translinear' proposed in [?].

shown in Fig. 12. It is shown that the two output stages depicted in Fig. 12(a,b) are not very useful in practice, whereas the third output stage is in fact a well-known circuit.

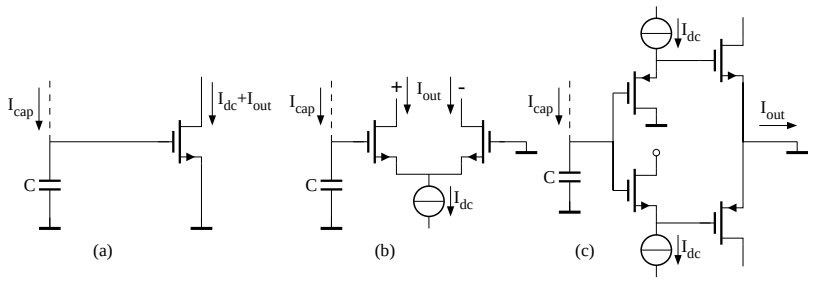

Figure 12: Generic dynamic voltage-translinear output structures.

5.1 $\sqrt{}$-domain output stage

The first DVTL output stage, the common-source stage, is characteristic for the class of $\sqrt{}$-domain filters [?, ?]. The capacitance current I_{cap} is given by:

$$I_{cap} = \frac{C}{\sqrt{2\beta}} \frac{\dot{I}_{out}}{\sqrt{I_{dc} + I_{out}}}. \tag{19}$$

where β is the MOS transconductance factor.

Equation (19) can be used to implement the derivative $\dot{I}_{out}$. A comparison of the log-domain and the $\sqrt{}$-domain output structures, based on eqns (4) and (19), shows that the latter class needs more circuitry to implement the derivative $\dot{I}_{out}$. Only four transistors are required to implement a log-domain low-pass filter. On the other hand, the implementation of the $\sqrt{}$-domain integrator described in [?] requires a square-rooting circuit and a multiplier. The multiplier again comprises two squaring circuits. The $\sqrt{}$-domain integrator published in [**?**] is a little more efficient as it requires a square-rooting circuit and only one squaring circuit. Nevertheless, it is clear that $\sqrt{}$-domain circuits require significantly more transistors than log-domain circuits.

5.2 Differential pair output stage

Even more hardware is required to implement a linear derivative $\dot{I}_{out}$ based on the differential pair output stage, see Fig. 12(b), as the capacitance current I_{cap} is now given by:

$$I_{cap} = \frac{C}{\sqrt{2\beta}} \dot{I}_{out} \left(\frac{1}{\sqrt{I_{dc} + I_{out}}} + \frac{1}{\sqrt{I_{dc} - I_{out}}} \right). \tag{20}$$

5.3 Analogue of the sinh output stage

More interesting is the output stage shown in Fig. 12(c), which is a direct translation of the sinh DTL output stage. The capacitance current is now given by:

$$I_{cap} = \frac{C}{4\sqrt{2\beta I_{dc}}}\dot{I}_{out}. \tag{21}$$

Note that I_{cap} and $\dot{I}_{out}$ are *linearly* related. Alternatively speaking, the large-signal V-I transfer function is linear. As a consequence, *no additional circuitry* is required to linearise this output stage. It is already linear. In addition, this stage allows a kind of class AB behaviour as the maximal current swing of I_{out} is four times as large as I_{dc}.

In fact, this output stage is a well-known circuit. Many, more or less similar, implementations have been published, see, e.g., [**?**, **?**, **?**, **?**]. In essence, the large-signal linear V-I relation is possible due to the fact that the square law MOS model is a polynomial function. An analogue principle for DTL circuits is fundamentally impossible since the exponential law is not a polynomial, but a transcendental function.

To conclude, we can state that the class of $\sqrt{}$-domain circuits is interesting from an academic point of view, but has little significance in practice. The same conclusion applies to DVTL circuits based on the differential pair output stage. Both the common-source and the differential pair output stage require a significant amount of additional hardware for linearisation purposes, which is not needed at all for the output stage shown in Fig. 12(c).

6 Synthesis methods

Several synthesis methods for translinear filters have been proposed in literature. This section gives an overview of these methods.

6.1 Voltage-mode synthesis

The design of first-order TL filters based on exponential transformations was introduced by Adams in [?]. Using a state-space approach, Frey was able to generalise this synthesis method to filters of arbitrary order [?]. In addition, Frey generalised this method to allow different exponential-like transformations [?,?]. The voltage-mode design procedure for TL filters begins with a state-space filter description:

$$CU_T\dot{\vec{I}}_x = \mathbf{A}\vec{I}_x + \mathbf{B}\vec{I}_u, \tag{22}$$

$$I_{out} = \mathbf{C}\vec{I}_x + \mathbf{D}\vec{I}_u, \tag{23}$$

where $\vec{I}_x$ is the state vector, $\dot{\vec{I}}_x = (\dot{I}_{x_1}, \ldots, \dot{I}_{x_n})^T$, $\vec{I}_u$ is the input vector, I_{out} the output signal, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are matrices.

Using a voltage-mode approach, the capacitance voltages V_{C_i} are used to represent the state of the filter. A state-space description in the voltage-domain is arrived at by applying exponential-like transformations to the states I_{x_i} and the input current $\vec{I}_u = [I_{in}]$. In general:

$$I_{x_i} = f_i(V_{C_i}), \qquad i \in [1, \ldots, n], \tag{24}$$

$$I_{in} = f(V_{in}), \tag{25}$$

where the functions f_i and f are strictly monotonic. Different choices can be made for the functions f_i:

$$I_{x_i} = I_{st} \exp \alpha V_{C_i}, \qquad I_{x_i} = I_{st} \tanh \tfrac{1}{2}\alpha V_{C_i}, \qquad I_{x_i} = I_{st} \sinh \alpha V_{C_i}, \tag{26}$$

where I_{st} and α are constants with dimensions [A] and [V^{-1}], respectively. These three functions comply with the classes of log-domain, tanh and sinh filters, respectively. For log-domain circuits, this mapping procedure is only valid if both I_{x_i}, $i \in [1, \ldots, n]$, and I_{in} are strictly positive. This restriction is satisfied by applying linear transformations to eqns (22) and (23), through trial-and-error, and/or by adding a dc input current I_{dc}, i.e., $\vec{I}_u = [I_{in}, I_{dc}]^T$.

As a result, a system of non-linear state-space equations results describing the TL filter in terms of voltages and exponential functions:

$$C\dot{V}_{C_i} = \sum_{j=1}^{n} \mathbf{a_{ij}} \frac{f(V_j)}{f'(V_i)U_T} + \mathbf{b_i} \frac{f(V_{in})}{f'(V_i)U_T}, \qquad i \in [1, \ldots, n], \tag{27}$$

where $\mathbf{a_{ij}}$ and $\mathbf{b_i}$ represent the elements of the matrices $\mathbf{A}$ and $\mathbf{B}$, and $f'(V_{C_i})$ is the first-order derivative of $f(V_{C_i})$ with respect to V_{C_i}.

Next, eqn (27) is interpreted as a set of nodal equations. The left-hand side of eqn (27) equals the current flowing through the capacitance C_i. Each of the terms on the RHS of eqn (27) takes the form of a controlled exponential-like transconductance. These controlled transconductances are mapped directly, or possibly after some rearranging, onto a circuit implementation. Often, standard building blocks are used to accomplish this. For example, E^+, E^-, T, S and $S2$ blocks are introduced in [?].

6.2 Component substitution

Another approach to the synthesis of TL filters is based on component substitution of prototype LC [?, ?, ?] or g_mC filters [**?**, **?**]. The general idea is to replace elements from a prototype filter by parts of TL loops. Within the general class of TL filters, only methods for the design of log-domain filters have been published.

All of these synthesis methods are based on the set-up depicted in Fig. 13, consisting of three essential parts. At the input, a single transistor is used to compress the input current I_{in}, resulting in a logarithmically related voltage V_{in}.

Next, this voltage is filtered by means of a so-called 'log-filter'. The resulting output voltage V_{out} is expanded exponentially, again by a single transistor, into the output current I_{out}.

Figure 13: Prevalent log-domain filter set-up.

Most of the design effort goes into the design of a 'log-integrator'. Basically, the first-order building blocks to be designed are an inverting and a non-inverting integrator, which are used next as substitutes for the integrator elements in the signal-flow graph. Within a higher-order filter network, the internal compression and expansion stages of the first-order building blocks cancel, and hence, these can be omitted, leaving only a single compression stage at the input and an expansion stage at the output of the complete filter. Linear losses, i.e., resistances, can simply be implemented by a dc current source in parallel with a capacitance, as explained in Section 4.

Application of these component substitution based design methods is simple. Yet, an important disadvantage seems that the designer cannot make any choices along the synthesis path. In general, for each LC or g_mC prototype filter, exactly one TL filter results. Therefore, the applicability of these methods is restricted.

6.3 Translinear synthesis

In [?], Seevinck used a current-mode approach to the design of two TL integrators. This current-mode synthesis methodology was generalised to filters of arbitrary order by the authors [?, ?]. A major advantage of this approach is that it fits directly onto the synthesis method for STL circuits described in [?]. Consequently, all existing theory and experience on STL circuits can be employed in the design of DTL circuits. The design trajectory of both STL and DTL circuits is depicted in Fig. 14, demonstrating the high level of similarity between these two classes of circuits.

In most cases, synthesis of a static or dynamic (transfer) function starts with a dimensionless equation. However, as soon as an electronic implementation has to be found, quantities are bound to certain dimensions. In the case of TL circuits, all signals and tuneable parameters, denoted by x_i, are transformed into currents by the equivalence relation:

$$x_i = \frac{I_{x_i}}{I_o}, \tag{28}$$

where I_o is a dc current. The dimensionless time τ has to be transformed into the time t with its usual dimension [s], using the equivalence relation given by:

$$\frac{\partial}{\partial \tau} = \frac{CU_T}{I_o}\frac{\partial}{\partial t}. \tag{29}$$

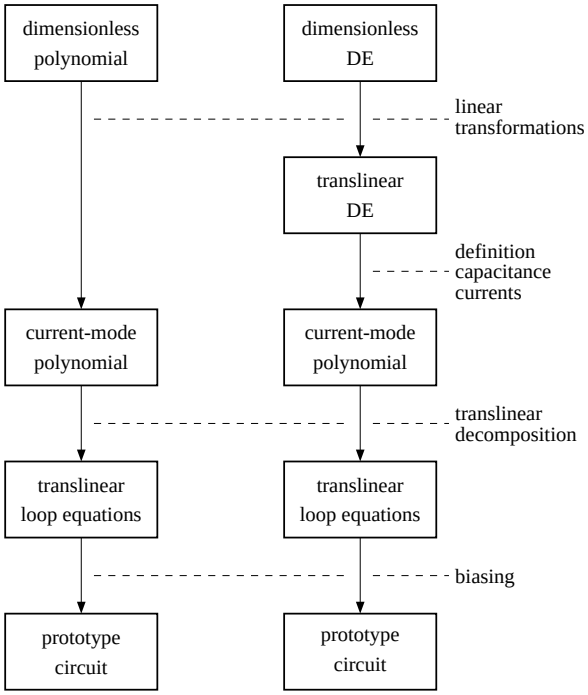

Figure 14: Synthesis path of (**a**) static and (**b**) dynamic translinear circuits.

Note that the presence of the current I_o in this expression explains the excellent linear tuneability of TL filters. Application of eqns (28) and (29) to a dimensionless linear state-space description yields (22) and (23).

Conventional TL circuits are described by multivariate polynomials, in which all variables are currents. The gap between these current-mode polynomials and the DE can be bridged by the introduction of capacitance currents. For, the DTL principle states that a derivative can be replaced by a product of currents. The capacitance currents, denoted by the vector $\vec{I}_{cap} = [I_{C_1}, \ldots, I_{C_n}]$, are introduced simply by defining them with the aid of eqn (6). In general, the capacitance currents are a function of $\vec{I}_x$ and $\dot{\vec{I}}_x$, i.e.:

$$\vec{I}_{cap} = \vec{I}_{cap}\left(\vec{I}_x, \dot{\vec{I}}_x\right). \tag{30}$$

To eliminate $\dot{\vec{I}}_x$ from eqn (22), (30) has to be solved to yield expressions for $\dot{\vec{I}}_x = \dot{\vec{I}}_x(\vec{I}_{cap}, \vec{I}_x)$. Substitution of these expressions in eqn (22) yields a set of *current-mode polynomials*. Equations (4), (16) and (18) are valid choices for the capacitance current definitions. Note that these equations are the current-mode equivalents of eqn (26). However, eqn (30) is more general than that. In general, each single capacitance current definition can depend on *all* state

variables. Two examples are:

$$I_{C_1} = CU_T \left(\frac{\dot{I}_{x_1}}{I_{x_1}} - \frac{\dot{I}_{x_2}}{I_{x_2}} \right), \tag{31}$$

$$I_{C_2} = CU_T \left(\frac{\dot{I}_{x_1}}{I_{x_1}} + \frac{\dot{I}_{x_2}}{I_{x_2}} \right), \tag{32}$$

which can be implemented by the output stages shown in Fig. 15. It is interesting to note that eqns (31) and (32) represent a more general class of TL filters as proposed in [?, ?]. An example of a second-order TL filter based on these generalised capacitance current definitions is described in [**?**].

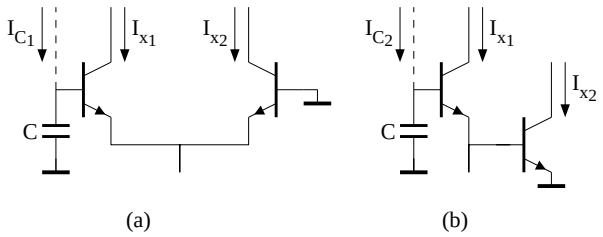

Figure 15: Two capacitance current definitions.

From this point on, the synthesis theory for STL circuits can be used, as illustrated by Fig. 14. The next synthesis step is TL decomposition. That is, the state-space polynomials have to be mapped on a set of TL loop equations, given by (2). A thorough treatment of the TL decomposition process can be found in [?].

The last synthesis step is biasing. The TL decomposition has to be mapped on a TL circuit topology and the correct collector currents have to be forced through the transistors. Biasing methods for bipolar all-NPN TL topologies are presented in [?]. Additional biasing methods include the use of (vertical) PNPs, compound transistors or (simple) nullor implementations. If subthreshold MOSTs are used, some additional possibilities are the application of the back gate [**?**, **?**] and operation in the triode region [?]. Once a proper biasing arrangement has been designed, the prototype circuit can be analysed for second-order effects. At this stage, an analysis method in the same domain is indispensable.

6.4 Synthesis based on the Bernoulli DE

The synthesis method for log-domain filters proposed in [?] follows a bottom-up approach. The design procedure is based on the generic circuit structure shown in Fig. 5. The analysis of the structure has shown that it is described by the state-space description (8). To synthesise a TL filter, eqn (8) is compared with a state-space description of the filter to be realised. This yields the necessary

form of the currents I_{u_k}. The task of the designer is to find ways to generate the currents I_{u_k} using conventional TL techniques.

7 State-of-the-art

Many skilfully designed DTL circuits have been presented in literature. Due to the limited space available, the state-of-the-art overview given in this section is limited to those designs that have been experimentally verified. To the authors' knowledge, this amounts to a total of eleven different TL filter designs [?,?,?,**?**, **?**,?,?,**?**,**?**,?,**?**] The specifications of these filters are summarised in Tables 1 and 2. Values between round brackets were calculated or estimated from the data presented in the cited publications.

Table 1: Class A log-domain filters.

	[?]	[?]	[?]	[?]	[?]	[?]
Process	Bipolar	0.8 μ BiCMOS	Bipolar	Bipolar	2μ CMOS	1 μ BiCMOS
Filter	LPF, 5	LPF, 4	BPF[Q:3.6–66], 5	LPF, 2	BPF, 2	LPF, 3
f_c [Hz]	40 k	100–10 k	125 M–430 M	1.6 k–8 k	75–15 k	10 k–10 M
DR [dB]	52	60	(55)	57	-	57
Total C [pF]	-	55	18	100	20	57
Power [W]	(25 m)	1 μ	81 m	6 μ	-	23 μ
Supply [V]	10	5	2.7	1	-	1.2

Table 2: Class AB translinear filters.

	[?]	[?]	[?]	[?]	[?]
Process	2 μ BiCMOS	1 μ BiCMOS	Bipolar	Bipolar	Bread-board
Filter	LPF, 3	LPF, 3	LPF, 1	APF, 2	LPF, 1
f_c [Hz]	10 k–100 k	10 k–15 M	1 k–8 k	155 k	1.6 k
DR [dB]	-	65	73	62	76
Total C [pF]	500	59	100	80	100
Power [W]	180 μ	65 μ	2 μ	2 μ	1 μ
Supply [V]	4	1.2	1	1.8	3.3

Probably the most important filter specification is the dynamic range (DR). Although the area of TL filters is still quite young, the results already compare well with the specifications that are typically obtained using g_mC filters, i.e., 40–70 dB [?]. Comparing the DR specification of the different TL filters is not easy, since the DR is influenced by many factors, e.g., power consumption and total capacitance. It is however interesting to note the difference in DR between class A and class AB operated TL filters. The filter specifications shown in Table 1 are obtained from log-domain filters operated in class A. Table 2 presents the results of class AB operated TL filters. The filters published in [?,?,?] are log-

domain filters; the filters published in [?, ?] are sinh filters. The class AB nature of the filters in Table 2 explains the structurally better DR specifications.

Application of the DTL principle is not limited to the implementation of linear filters. Non-linear DEs can be realised just as well. As of today, the list of experimentally verified designs is still quite short. Only three designs are known to the authors: an oscillator [?], an RMS-DC converter [?] and a PLL [?, ?]. The interested readers are referred to these papers for measurement results.

With the increasing number of researchers of DTL circuits, we can expect to see more and more realisations, both for linear and non-linear applications. It will be interesting to see to what extend the specifications obtained thus far can be improved in the future.

8 Conclusions

Dynamic translinear circuits constitute an exciting new approach to the integration of analogue signal processing functions. These circuits might prove to be the best approach to face the dynamic range limitations, conventional integrated circuits are facing due to low-power, low-voltage and high-frequency demands. This field is receiving increasing interest and encouraging results have been obtained thus far. The future will show to what extent these results can be improved.