Analysis of Noise in Translinear Filters

J. Mulder, M.H.L. Kouwenhoven, W.A. Serdijn, A.C. van der Woerd, and A.H.M. van Roermund
Electronics Research Laboratory / DIMES - Delft University of Technology
Mekelweg 4, 2628 CD Delft, The Netherlands, E-mail: j.mulder@et.tudelft.nl

Abstract

To describe the effects of noise in translinear filters, large-signal equations have to be used, due to the internal non-linearities and the non-stationary nature of the noise sources. In this paper, a noise analysis method is proposed, which takes into account the non-linear and non-stationary aspects. As an example, the signal × noise intermodulation is calculated for a class A and a class AB operated log-domain filter.

I Introduction

Translinear circuits are experiencing a strong revival due to the recent generalisation to frequency-dependent transfer functions [1–3]. Whereas the conventional class of 'Static TransLinear' (STL) circuits implements static, i.e. frequency-independent, functions, the class of 'Dynamic TransLinear' (DTL) circuits [4] can be used to implement Differential Equations (DE).

Although the overall transfer function of a TL filter is linear, the internal non-linear behaviour complicates the analysis of noise, as it results in intermodulation between signals and noise. Furthermore, the non-stationary nature of the transistor noise sources, treated in Sec. II, adds to the complexity of the situation.

To include the above effects, large-signal equations have to be employed to describe the operation of TL filters in the presence of noise. In [5], a large-signal analysis method for DTL circuits was proposed. In this paper, a non-linear analysis method for noise in TL filters is developed, based on the method presented in [5]. The actual noise analysis method is described in Sec. III. In Secs IV and V, its application is demonstrated for a log-domain filter operated in class A and class AB, respectively.

II Transistor noise sources

The internal noise of TL filters is due to the noise mechanisms present in the transistors comprising the circuit. Therefore, it is important to investigate these noise sources. Here, the discussion is limited to the bipolar transistor, as this device has been used in the majority of DTL circuits published to date.

The noise behaviour of the bipolar transistor is mainly characterised by four statistically independent noise sources. First, the collector current I_C introduces a shot noise source i_C. The double-sided power spectral density function S_{i_C} of this white noise source is given by:

$$S_{i_C}(\omega, t) = qI_C(t), \tag{1}$$

where q is the unity charge. Due to the signal-dependence of the transistor currents in TL circuits, S_{i_C} is in general non-stationary.

The second noise source is the shot noise accompanying the base current. It is often considered in conjunction with the $1/f$ noise, the third noise source. In TL circuits, these two noise sources are almost always negligible [6].

The last noise source v_{R_B} originates from the base resistance R_B. The voltage-mode description of v_{R_B} does not comply very well with the well-known current-mode approach to TL circuits. We can advantageously transform v_{R_B} into a noise current i_{R_B} in parallel with i_C, using the (small-signal) transconductance g_m of the transistor. The resulting power spectral density function $S_{i_{R_B}}$ is given by:

$$S_{i_{R_B}}(\omega, t) = qI_C(t)\,\frac{2R_B I_C(t)}{U_T}. \tag{2}$$

This transformation is allowed since v_{R_B} is much smaller than the thermal voltage U_T. Further, the signal-dependence of g_m, which is a source of signal × noise intermodulation, has to be taken into account.

Comparing eqns (1) and (2), we can conclude that v_{R_B} is negligible when the transistor is operated at low current levels, where $I_C \ll \frac{1}{2}U_T/R_B$. Conversely, at high current levels, where $I_C \gg \frac{1}{2}U_T/R_B$, v_{R_B} is the dominant source of noise.

III Noise analysis method

In [5], a general analysis method for DTL circuits was proposed. Since this method uses large-signal equations to account for the internal non-linearities of TL filters, it can be used as the basis for a non-linear noise analysis procedure. The sequential stages of the noise analysis process are explained in this section. It is assumed that all second-order effects, e.g. finite current gain (β_F), have a negligible influence on the noise behaviour. This is however not a fundamental limitation of the proposed method.

• The first step in the analysis of both STL and DTL circuits is the determination of the transistor currents, which follow from the currents applied to the nodes of the TL core, through application of Kirchhoff's Current Law (KCL). Since both noise sources i_C and i_{R_B} of the bipolar transistor are modelled by currents, see eqns (1)-(2), all internal noise sources of the TL circuit can be incorporated directly into the KCLs. Note that the power spectra of all noise sources are known once the collector currents have been determined, see eqns (1)-(2).

• Next, the independent TL loops have to be identified. This step is again equivalent for STL and DTL circuits. Once the TL loops are found, the TL loop equations can be used to describe the circuit in terms of the familiar products of collector currents [6, 7].

As an example consider the four-transistor TL loop depicted in Fig. 1, where it is assumed that the transistors Q_1 through Q_4 are somehow biased at the currents I_1-I_4. Including the noise sources i_1-i_4, the TL loop equation is given by:

$$(I_1 + i_1)(I_3 + i_3) = (I_2 + i_2)(I_4 + i_4). \qquad (3)$$

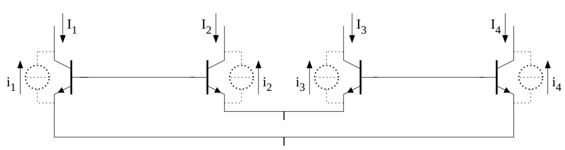

Figure 1: A second-order translinear loop in the presence of noise.

• In case of DTL circuits, the collector currents are linear combinations of the input, output, noise and capacitance currents. The latter type of currents accounts for the frequency-dependent behaviour of DTL circuits. To obtain the DE, describing the transfer function of the circuit, from the TL loop equation, the capacitance currents have to be eliminated. Expressions for the capacitance currents can be derived easily from the collector currents [5]. Since the collector currents include noise sources, the capacitance current expressions will introduce time derivatives of noise sources. These derivatives can be considered as additional, yet correlated, noise sources. Substitution of the capacitance current expressions in the TL loop equation directly yields the DE.

• As the noise currents can be assumed to be small with respect to the collector currents, two approximations can be made to simplify the equations. First, products of noise sources can be neglected. For example, in eqn (3), the products $i_1 i_3$ and $i_2 i_4$ can be eliminated.

Second, a multi-dimensional first-order Taylor series approximation with respect to all noise sources can be made. It is important to note that this Taylor series expansion preserves both the signal-dependence of the noise sources and the signal $\times$ noise intermodulation.

The DE thus obtained describes the influence of the internal noise sources in the time-domain. It can be divided into the noise-free DE, and a part comprising only noise. Conventional noise analysis techniques can now be used to calculate the equivalent output noise and the signal-to-noise-ratio (SNR).

• The next step is to find the autocorrelation function of the noise expression. Statistically independent noise sources can be examined separately. Naturally, correlated noise sources have to be dealt with simultaneously.

• The autocorrelation function is related to the power spectral density function via the Wiener-Khintchine theorem, which can be generalised to non-stationary noise sources [8]. Consequently, the Fourier transformation has to be applied to calculate the power spectrum of the noise, see e.g. [9].

• The resulting spectrum is in general non-stationary. As it is difficult to interpret the SNR of a circuit with a non-stationary noise spectrum, it is convenient to use a stationary interpretation to define the SNR. A logical choice is to use the time-averaged noise spectrum.

IV A class A operated translinear filter

As an example of the proposed noise analysis method, we calculate the SNR of the first-order low-pass TL filter shown in Fig. 2, which is presumed to operate in class A. Basically, the filter consists of a second-order folded TL loop, comprising four transistors Q_1-Q_4 and a capacitance C. The figure includes the four collector shot noise sources i_1-i_4. Assuming low-power operation, i_1-i_4 dominate over the noise due to the base resistances.

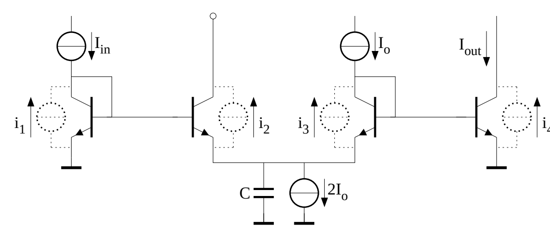

Figure 2: Noise in a translinear first-order low-pass filter.

In the ideal situation, the capacitance current I_{cap} is only determined by the collector current of Q_4 [2, 5]. Hence, it is convenient to transform i_3 to an equivalent noise source $i_{3,eq}$ in parallel with i_4 before applying the method outlined in the previous section. Thus, i_3 is transformed to an equivalent noise source $i_{3,eq} = i_3 \cdot I_{out}/I_o$ at the output of the filter.

Based on the KCLs, the four collector currents are easily found from Fig. 2, and consequently the TL loop equation can be derived, which yields:

$$(I_{in} + i_1)I_o = (I_o + I_{cap} + i_2)(I_{out} + i_{3,eq} + i_4). \qquad (4)$$

The expression for I_{cap} in the presence of noise is derived from the collector current of Q_4, and is given by [5]:

$$I_{cap} = CU_T \frac{\frac{\partial}{\partial t}(I_{out} + i_{3,eq} + i_4)}{I_{out} + i_{3,eq} + i_4}. \qquad (5)$$

Substitution of this expression in eqn (4) and elimination of products of noise currents yields the DE describing the TL filter:

$$\frac{CU_T}{I_o}\dot{I}_{out} + I_{out} - I_{in} = \frac{CU_T}{I_o}\frac{\partial}{\partial t}(i_{3,eq} + i_4) + i_1 + i_2\frac{I_{out}}{I_o} + i_{3,eq} + i_4. \quad (6)$$

The left-hand side of this equation gives the noise-free DE, whereas the right-hand side consists only of noise.

A closer examination of eqn (6) reveals that the effect of the respective noise sources on the equivalent output noise is different. i_1 is in parallel with I_{in}, and hence, it is low-pass filtered by the linear TL filter. $i_{3,eq}$ and i_4 are already at the output, and therefore result in white noise at the output. Finally, i_2 is modulated by the output current I_{out}, and consecutively filtered by the TL filter. Note that these results derived from eqn (6) comply with the position of the noise sources shown in Fig. 2.

The next analysis step, the calculation of the autocorrelation functions of the four noise sources, is trivial for i_1 and i_4, see e.g. [9]. The resulting power spectra equal $S_{i_1}(\omega, t) = qI_{in}$ and $S_{i_4}(\omega, t) = qI_{out}$ at the input and output of the filter, respectively.

The autocorrelation function $R(i_2 \cdot I_{out})$ of the term $i_2 \cdot I_{out}$ equals the product of the autocorrelation functions R_{i_2} and $R_{I_{out}}$ of i_2 and I_{out}, respectively, since, for $i_2 \ll I_{out}$, i_2 and I_{out} can be considered as being uncorrelated: $R(i_2 \cdot I_{out}) = R_{i_2} \cdot R_{I_{out}}$. A similar relation holds for the autocorrelation function of the term $i_3 \cdot I_{out}$.

Application of the Fourier transformation to the product $R_{i_2} \cdot R_{I_{out}}$, to obtain the corresponding power spectrum $S(i_2 \cdot I_{out})$, results in a convolution in the frequency domain, see e.g. [9]:

$$S(i_2 \cdot I_{out})(\omega, t) = \frac{1}{2\pi} S_{i_2}(\omega, t) * S_{I_{out}}(\omega, t), \quad (7)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{i_2}(y, t) S_{I_{out}}(\omega - y, t)\mathrm{d}y. \quad (8)$$

Since i_2 is a white noise source, the integral can be easily solved and yields:

$$S(i_2 \cdot I_{out})(\omega, t) = S_{i_2}(\omega, t) \cdot P_{I_{out}}, \quad (9)$$

where $P_{I_{out}}$ denotes the output power. A similar relation holds for the spectrum $S(i_3 \cdot I_{out})$.

Now, S_{i_1} and $S(i_2 \cdot I_{out})$, which are still at the input of the TL filter, have to be transformed to the output. Due to the non-stationary nature of the noise sources, in principle, a two-dimensional Fourier transformation can be used to calculate the equivalent non-stationary output noise spectrum [9]. However, since we are only interested in the average noise spectrum to calculate the SNR, we can exchange the operations of linear filtering and time-averaging. As a result, the output noise average power spectrum $\overline{S_T}(\omega)$ is found to be:

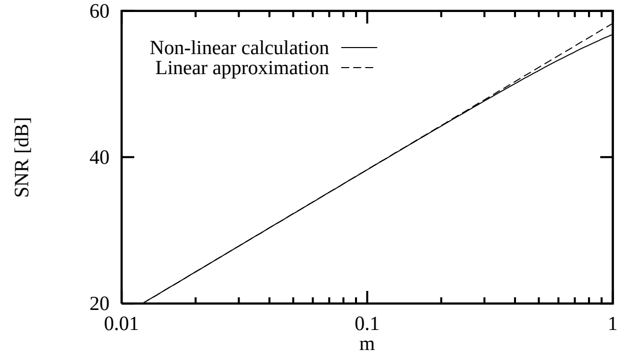

Figure 3: Signal-to-noise-ratio for a TL filter operated in class A.

$$\overline{S_T}(\omega) = q\left(\overline{I_{in}} + \frac{\overline{I_o + I_{cap}}}{I_o^2}\right)|H(\omega)|^2 + \frac{qP_{I_{out}}}{I_o} + q\overline{I_{out}}, \quad (10)$$

where $H(\omega)$ represents the filter transfer function, and $\overline{(\cdot)}$ denotes the time average.

Since I_{cap} cannot contain a dc component, $\overline{I_o + I_{cap}}$ equals I_o. When the filter, shown in Fig. 2, is operated in class A, $\overline{I_{in}}$ and $\overline{I_{out}}$ are both equal to the dc bias current I_{dc}.

Using these facts, and applying the noise bandwidth of the filter to $i_{3,eq}$ and i_4 as well, the SNR of the class A TL filter can now be determined. Considering a sinusoidal input current within the pass-band of the filter, $I_{in} = I_{dc}(1 + m\sin\omega t)$, where m is the modulation index, the SNR as a function of m plotted in Fig. 3 is obtained. In this figure, $C = 10$ pF, $I_{dc} = 5\mu$A and $I_o = 1\ \mu$A.

Since $m < 1$ for a class A filter, the influence of signal $\times$ noise intermodulation is very small. The difference between the non-linear calculation and the linear approximation, shown for reference in Fig. 3, equals only 1.51 dB for $m = 1$. Hence, for class A TL filters, the noise floor in the absence of any signals can be used as a very good estimate of the noise. This is not the case in class AB filters, as shown in the next section.

V A class AB operated translinear filter

The noise of TL filters operated in class AB is much more interesting as the signal $\times$ noise intermodulation effects are more pronounced here. A possible set-up for class AB operation is shown in Fig. 4 [10]. At the input, the input signal is first split into two strictly positive parts, I_{in1} and I_{in2}, by a current splitter. The difference of I_{in1} and I_{in2} equals the original input signal I_{in}. Different current splitters can be used, which implement a different relation between I_{in1} and I_{in2}. Common types are the geometric mean splitter, $I_{in1}I_{in2} = I_{dc}^2$, and the harmonic mean splitter, $I_{in1}I_{in2} = \frac{1}{2}(I_{in1} + I_{in2})I_{dc}$. Next, I_{in1} and I_{in2} are linearly filtered by separate signal paths.

At the output, I_{out} is obtained by subtraction of the two currents I_{out1} and I_{out2}.

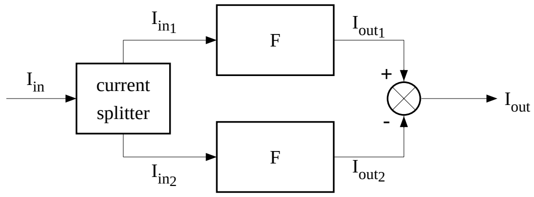

Figure 4: Set-up for class AB operation.

Class AB operation of the filter shown in Fig. 2 is obtained by including it in the set-up shown in Fig. 4. Using the proposed noise analysis method, it can be shown that the noise contribution of the current splitter itself is negligible. Consequently, for class AB operation, the noise of the first-order TL filter is again characterised by eqn (10), except for a factor two to account for the two signal paths. In eqn (10), I_{out} must be replaced by I_{out1} and I_{in} by I_{in1}.

The average noise spectrum $\overline{S_{\mathcal{T}}}$ is determined by $\overline{I_{in1}}$ and $\overline{I_{out1}}$. These averages depend to some extend on the splitter being used.

Now, calculation of the SNR results in the graph shown in Fig. 5. In this figure, $C = 10$ pF, and $I_{dc} = I_o = 1\mu$A. For low values of m, the noise level is constant and the SNR increases 20 dB/decade. However, for values of m between 1 and 10, the non-linear noise starts to dominate and the SNR saturates to a value of 62.1 dB.

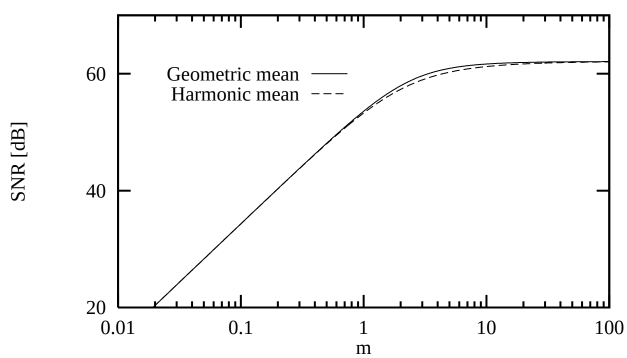

Figure 5: Signal-to-noise-ratio for different current splitters.

In Fig. 5, the SNR is plotted both for the geometric mean and the harmonic mean current splitter. The SNR is identical for low and high values of m, whereas for intermediate values, application of the harmonic mean splitter introduces some extra noise. The maximum difference is however only 0.71 dB.

A major advantage of the proposed method is its comprehensiveness. For example, eqn (10) incorporates the different influence of in-band versus out-of-band signals being processed by the filter. Figure 6 displays the noise power spectrum for a sinusoidal input signal, with $m = 10$, at the frequencies $\omega = [\frac{1}{10}, 1, 10]\omega_c$, ω_c being the cut-off frequency of the filter. For reference, also the noise level in the absence of any signals is depicted.

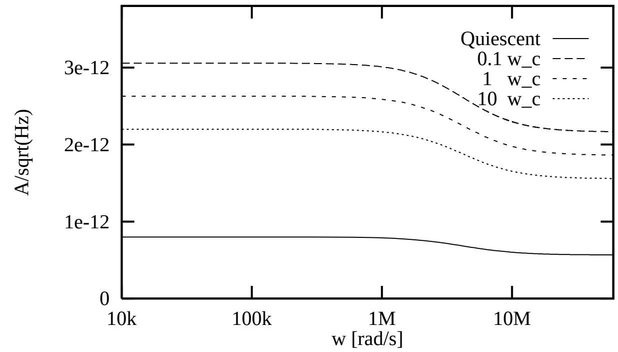

Figure 6: Influence of signal $\times$ noise intermodulation on the noise spectrum.

VI Conclusions

In translinear filters, signal $\times$ noise intermodulation originates from the internal multiplicative non-linear behaviour. By including the transistor noise sources in a large-signal analysis method, calculation of the non-linear noise effects is facilitated. Application of the proposed noise analysis method has been demonstrated for class A and class AB operated TL filters.

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