MATCHING OF LOW-NOISE AMPLIFIERS
AT HIGH FREQUENCIES

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ABSTRACT

Higher transistor transition frequencies, lower supply voltages and smaller physical dimensions are, nowadays, general trends in the semiconductor industry. Operating at lower supply voltages often results in a low-power design, smaller dimensions allow the use of a large number of transistors and a high transition frequency (f_T) opens the way to design of amplifiers with ever-higher gains and lower noise figures. However, such trends make the use of some circuit topologies questionable. Take the inductively-degenerated low-noise amplifier, arguably the most-widely used RF amplifier topology, requiring an impractical inductance in the order of *pH* for a simultaneous input-power match at high f_Ts. Therefore, a conceptual change in a design approach has resulted in transformer-feedback degenerated low-noise amplifier, presented in this paper. By controlling the coupling coefficient, the power match is possible even for the highest values of f_T, with practical values of primary inductance for the transformer. The analysis gives full insight into the performance of the newly introduced transformer-feedback degenerated low-noise amplifier scheme.

1. INTRODUCTION

Enabled by the improvements in the semiconductor technology, the tendency of moving to higher transition frequencies, lower supply voltages and smaller dimensions must be directly followed by certain changes in the design philosophy, as well. As on one hand, due to their smaller dimensions, there is an option of integrating a large number of Si-based components on the same chip, there is, on the other hand, a limitation of stacking them due to lower supply voltages. From the perspective of obtaining a lower noise figure or a higher power gain of the amplifiers, the increase in the transition frequency is fully beneficial. However, the necessity of a change in the design approach and the existing circuit topologies, for the same reason, being high f_T, seems to be rather unnoticeable.

Take the inductively-degenerated bipolar low-noise amplifier (LNA) [1], which due to its features of simultaneous power-noise match and superior noise performance is nowadays the most-widely used RF LNA topology. In this scheme, a 50Ω input impedance, that is set by means of a certain value of the inductance in the emitter of the LNA's input transistor, is impossible to achieve as technology (f_T) improves. For example, for f_T=150GHz, a matching-inductance of 0.05nH is required, that cannot be safely designed and integrated on chip.

Applying *transformer-feedback* degeneration to the LNA, as proposed in this paper and in contrast to inductive degeneration, it is possible to achieve input match, even at very high operating frequencies and accordingly required high f_T's. Controlling the amount of feedback through the transformer's coupling coefficient, the real part of input impedance can be set to 50Ω and the imaginary part to zero, while at the same time a larger and *design-safer primary inductor* of the implemented transformer can be used. What is more, this technique offers the possibility for the low-noise amplifiers to achieve simultaneous match of both the real and the imaginary part of the input impedance in an orthogonal way.

The paper is divided into six sections. A novel high-frequency matching model is presented in Section 2. To the authors' knowledge, the most comprehensive performance parameters characterization is presented; a power-gain analysis in Section 3, and a noise-figure analysis in Section 4. An over-all example, comparing the performances of the inductively and transformer-feedback degenerated LNA, is presented in Section 5. Finally, the conclusions are drawn in Section 6.

2. HIGH F_T MATCHING MODEL

Even though the inductively-degenerated (ID) low-noise amplifier [1] is effective in realizing power-noise match, due to the fact that optimum noise resistance and device input resistance can be adjusted independently, in order to be beneficially employed, as technologies move to ever-higher transition frequencies, a certain adaptation in the topology is required. At high f_Ts, the most distinguished property of ID LNA, being the input-power match, relying on the series feedback via the inductor in the emitter of the amplifier input transistor, is lost, as the required inductance appears to be so small, that it can not be safely designed.

Therefore, the transformer-feedback degenerated (TFD) LNA is introduced, maintaining all the striking properties of the conventional ID LNA, even at the highest f_Ts. A schematic of transformer-feedback degenerated low-noise amplifier is shown in Fig. 1, without a complete biasing.
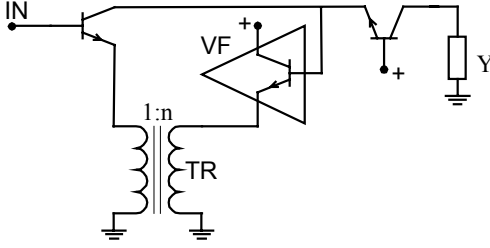
Fig. 1 Transformer-feedback degenerated LNA.

This amplifier topology is a traditional cascode configuration, with the addition of the feedback around the input transistor, that is realized by means of a voltage-follower (VF), in its simplest form a single transistor in a common-collector configuration and a transformer TR.

To express the power-matching condition of the TFD LNA, let us first calculate the input impedance that is to be matched to the source of the amplifier. For that purpose, we will refer to the equivalent circuit of the amplifier shown in Fig. 1, with the addition of a first order transformer [2]. This is shown in Fig. 2,
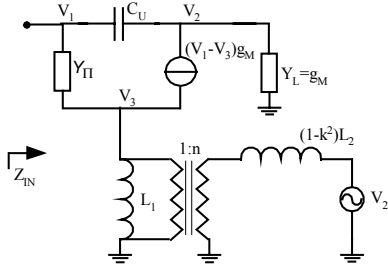

Fig. 2 Detailed schematic of the TFD LNA.

where Y_Π is the base-emitter admittance, dominated by capacitance C_Π for high frequencies, C_U is the Miller capacitance, g_m the transconductance of the bipolar transistor, Y_L the input admittance of the following stage, in this case the input admittance of the common base transistor, L_1 and L_2 are the transformer primary and secondary inductors, n is the transformer turn ratio, and k is the coupling factor between the transformer inductors.

Applying Kirchoff's current law to the circuit of Fig. 2, with the assumption that at the frequency of interest $sC_U << g_m$ and $sC_U << Y_L = g_{m2} = g_m$, the equivalent input admittance becomes:

$$Y_{IN} = Y_\Pi \cdot f(Y_1, Y_2) + sC_U[1 + g_m / Y_L \cdot f(Y_1, Y_2)] \qquad (1)$$

where $f(Y_1, Y_2)$ is the so-called *feedback function* equal to:

$$f(Y_1, Y_2) = 1 - V_3 / V_1 \qquad (2)$$

Now, the input impedance can be estimated, simply, from Eq. (1) that accounts for the feedback over the capacitance C_U, and the function f, that however can be calculated without taking into account the Miller effect.

If the primary and the secondary inductor of the transformer TR are, by definition, related as $L_2/L_1 = n^2/k^2$, it is rather straightforward to calculate the voltage transfer from node V_1 to node V_3, and subsequently the function $f(Y_1, Y_2)$ as:

$$f(Y_1, Y_2) = \frac{Y_1 + n^2 Y_2}{Y_\Pi + Y_1 + n^2 Y_2 + g_m(1 \pm nY_2 / Y_L)} \qquad (3)$$

with $Y_1 = 1/sL_1$ and $Y_2 = 1/s(1 - k^2)L_2$. Depending on the orientation of the transformer, the feedback can be either negative or positive, which is the origin of the $\pm$ sign in Eq. (3).

Rearranging Eqs. (1) and (3), the equivalent input impedance, for slightly positive feedback, can be given a form:

$$Z_{IN} = \omega_T(1 - k^2)L_1 + j\left[\omega_0(1 - k^2)L_1 + \frac{\omega_T}{\omega_0}\frac{k^2}{n}\frac{1}{g_m} - \frac{\omega_T}{\omega_0}\frac{1}{g_m} \right] \qquad (4)$$

where it is assumed that $C_\Pi/C_U >> 1$ and $1/\omega_0 C_U >> \omega_T(1 - k^2)L_1$, with $\omega_T = 2\pi f_T$ and $\omega_0 = 2\pi f_0$ being the corresponding transition and operating angular frequencies.

Now, the condition for the match of the real part of the input impedance to a source impedance R_S, is derived from Eq. (4) as:

$$2\pi f_T(1 - k^2)L_1 = R_S \qquad (5)$$

which resembles the ID case. However, it is now possible to rely on a design-safe primary inductance L_1, as a result of a possible larger, for a *very high f_T, close to a unity value, coupling coefficient k.*

On the other hand, setting the imaginary part of the input impedance to zero is facilitated, as compared to the ID, simply because the feedback-resulting inductance $(\omega_T/\omega_0)(1/g_m)(k^2/n)$, recognized in Eq. (4), enables the immediate cancellation of the capacitance C_Π. The condition to be satisfied is again derived from Eq. (4), by setting imaginary part to zero, and has a form:

$$\frac{k^2}{n} = g_m\left[\frac{1}{g_m} - (\frac{\omega_0}{\omega_T})^2 R_S \right] \qquad (6)$$

Here, the full matching is achieved without a need for a *lossy* input inductor, being responsible for the imaginary part cancellation of the input impedance in the ID LNA scheme.

For example, as for a 50Ω input impedance match, using technology with the $f_T = 100$GHz, an inductance of 0.075nH is required for the ID LNA, in case of TFD LNA with a coupling coefficient $k = 0.9$, a reasonable primary inductance of 0.39nH is required.

Finally, the properties of high f_T match as well as orthogonal input-power match, promote this topology in a good candidate for RF applications.

3. GAIN MODEL

Not only can the feedback function f be used for the estimation of the input impedance, but also it can be used for the calculation of the other LNA performance parameters, among them being power gain and effective transconductance.

Accordingly, from the equivalent circuit model of the TFD amplifier, shown in Fig. 2, and from Eqs. (3) and (4), the effective transconductance G_{EFF} and power gain PG, both with respect to the input of the amplifier, can be expressed as:

$$G_{EFF} = -g_m\frac{(1 - V_3/V_1)}{1 + Y_\Pi/Y_S(1 - V_3/V_1)} = -g_m\frac{f(Y_1, Y_1)}{1 + Y_\Pi/Y_S f(Y_1, Y_1)} \qquad (7)$$

$$PG = \frac{g_m^2|1 - V_3/V_1|^2}{Y Y_S|1 + Y_\Pi/Y_S(1 - V_3/V_1)|^2} = \frac{g_m^2|f(Y_1, Y_2)|^2}{Y Y_S|1 + Y_\Pi/Y_S f(Y_1, Y_2)|^2} \qquad (8)$$

where Y_S is the source admittance.

However, for the full input power-match, i.e., the real part of the input impedance is equal to the source resistance R_S, and the imaginary part is zero, and with the assumption that source and load impedances are the same, $1/Y = R = R_S$, the effective

transconductance and power gain of the TFD LNA, can finally be expressed as:

$$G_{EFF} = -\frac{g_m}{Y_\Pi} \frac{1}{Z_{IN} + Z_S} = -\frac{1}{2R_S} \frac{\omega_T}{\omega_0} \tag{9}$$

$$PG = \left|\frac{g_m}{Y_\Pi}\right|^2 \frac{Z \, Z_S}{(Z_{IN} + Z_S)^2} = \frac{1}{4}(\frac{\omega_T}{\omega_0})^2 \tag{10}$$

When compared to the same performance parameters of the, so far, "best" ID LNA topology [3], one can notice that obtained results are the same. This implies that due to better matching scheme, TFD topology may become the leading one.

4. NOISE MODEL

Defined as the ratio of the equivalent input voltage noise spectral density of the amplifier and voltage noise spectral density of the source, the noise figure is one of the most important specification parameters of RF front-end systems, as it directly sets the overall front-end sensitivity. Due to its importance, in the coming sub-sections we will calculate all the noise-related parameters, such as: noise-figure (NF), optimum noise resistance ($R_{S,OPT}$) and optimum noise-figure (NF_{OPT}) as well as optimum-minimum noise-figure ($NF_{OPT,MIN}$).

4.1. Noise-figure

In order to find the expression for the noise figure NF, the equivalent voltage noise spectral density of the TFD LNA, shown in Fig. 1, must be calculated first. For that purpose, the amplifier noise model, with corresponding main noise sources, is shown in Fig. 3.
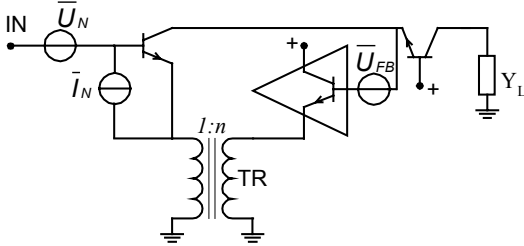

Fig. 3 Noise model of TFD LNA.

Applying the Blakesley transformation to the voltage noise sources and splitting the current noise sources, while at the same time keeping their orientation, the equivalent voltage noise at the input of the LNA is found as:

$$\overline{U}_{N,EQ} = \overline{U}_N + (Z_S + Z_E)\overline{I}_N + \frac{1}{n}\overline{U}_{FB} \tag{11}$$

where $\overline{U}_N$ and $\overline{I}_N$ the equivalent input-referred noise sources of the common-emitter transistor, $\overline{U}_{FB}$ voltage noise source at the input of the LNA's second, common-base, transistor, $Z_S = R_S$ and $Z_E = s(1-k^2)L_1 = sL_E$. These noise sources can be expressed as:

$$\overline{U}_N = \overline{U}_B - B_N \overline{I}_C + (r_{B1} + r_{E1})(\overline{I}_B - D_N \overline{I}_C) \qquad \overline{I}_N = \overline{I}_B - D_N \overline{I}_C \tag{12}$$

$$\overline{U}_{FB}^2 = 4KT(r_{B2} + r_{E2} + \frac{1}{2g_m} + \frac{1}{g_m^2 R_L}) \tag{13}$$

$$\overline{I}_B^2 = 2qI_B \quad \overline{I}_C^2 = 2qI_C \quad \overline{U}_B^2 = 4KT(r_B + r_E) \tag{14}$$

where $\overline{I}_B$ is the base current shot-noise, $\overline{I}_C$ the collector current shot-noise, $\overline{U}_B$ the base and the emitter resistance ($r_B + r_E$) thermal noise and $B_N = -1/g_m$ and $D_N = -(1/\beta_F + j\omega/\omega_T)$ transistor's transmission parameters, with $B_F \cong \beta_F$ being DC and AC transistor's current gain factors, respectively.

Now, with the aid of Eqs. (11)-(14), the noise figure is found to be:

$$NF = 1 + \alpha\delta + \frac{g_m}{2}\delta R_S + \frac{1 + 2\alpha + \alpha^2\delta + \delta L_G^2 - L_G\omega/2\omega_T}{2g_m R_S} \tag{15}$$

$$\delta = \frac{1}{\beta_F} + (\frac{\omega}{\omega_T})^2 \quad r_{EF} = r_{B1} + r_{E1} + \frac{1}{n^2}(r_{B2} + r_{E2} + \frac{1}{2g_m} + \frac{1}{g_m^2 R_L}) \tag{16}$$

$$\alpha = r_{EF}g_m(1 + 1/\beta_F) \qquad L_G = g_m\omega L_E(1 + 1/\beta_F) \tag{17}$$

4.2. Optimum noise parameters

Referred to as the optimum noise resistance, it is the source resistance necessary to achieve noise matching and, accordingly, optimum noise figure at the desired frequency. The expression for the *optimum noise resistance* $R_{S,OPT}$, can be found after solving the differential equation $dNF/dR_S = 0$. Accordingly, from Eq. (15), the most comprehensive form of the $R_{S,OPT}$ is:

$$R_{S,OPT} = \frac{1}{g_m}\sqrt{\frac{1 + 2\alpha + \frac{(1+\alpha)^2}{\beta_F} + \alpha^2(\frac{\omega}{\omega_T})^2 + \delta L_G^2 - \frac{\omega}{2\omega_T}L_G}{\delta}} \tag{18}$$

Assuming that current gain is larger than one, $\beta_F >> 1$, Eq. (18) simplifies to:

$$R_{S,OPT} = \frac{1}{g_m}\sqrt{(r_{EF}g_m)^2 + \frac{1 + 2r_{EF}g_m}{1/\beta_F + (\omega/\omega_T)^2}} \tag{19}$$

On the other hand, with the aid of Eqs. (15) and (18), the *optimum noise-figure* can be expressed as given in Eq. (20):

$$NF_{OPT} \cong 1 + \alpha\delta + \sqrt{\delta\left[1 + 2\alpha + \frac{(1+\alpha)^2}{\beta_F} + \alpha^2(\frac{\omega}{\omega_T})^2\right]} \tag{20}$$

which after the simplifications, $\beta_F >> 1$ and $\alpha\delta << 1$ reduces to:

$$NF_{OPT} \cong 1 + \sqrt{(1 + 2r_{EF}g_m)[\frac{1}{\beta_F} + (\frac{\omega}{\omega_T})^2]} \tag{21}$$

At this point, one should note that all the expressions for the TFD are the same as those for the ID, with the only difference in r_{EF}, which is in case of ID just the sum of the base and the emitter resistances of the first transistor.

4.3. Optimum-minimum noise-figure

After the noise-matching source resistance and optimum noise-figure are found, the minimum of the optimum noise figure is obtained for a certain biasing condition, finally resulting in the *optimum-minimum noise figure*. This condition can be found from the equation $dNF_{OPT}/dg_m = 0$, solving it for the biasing point g_m.

With the aid of Eq. (20), the condition for the minimum of the optimum noise-figure, after fair approximations, equals:

$$g_{m,OPT} \cong \omega C_\Pi \sqrt{\beta_F\left[1 + \frac{1}{r_{EF}g_{m,OPT}}\right]} \tag{22}$$

where it becomes obvious that the solution for the

transconductance g_m, can be found only iteratively. In such an approach, the solution for the g_m,

$$g_{m,OPT,START} = \omega C_\Pi \sqrt{\beta_F} \quad (23)$$

that is given in [4] as the final, is here considered to be only the starting solution for Eq. (22).

Now, the *optimum-minimum noise resistance*, after substituting (22) into (18), can be written as:

$$R_{S,OPT,MIN} = \frac{1}{g_{m,OPT}} \sqrt{\frac{1}{\beta_F}(1+\alpha)} = \frac{1}{g_{m,OPT}} \sqrt{\frac{1}{\beta_F}(1+r_{EF}g_{m,OPT})} \quad (24)$$

Finally, substituting the condition (22) in the expression (20) for the optimum noise figure, its *minimum* becomes:

$$NF_{OPT,MIN} = 1 + \frac{\alpha(1+2\alpha)}{\beta_F(1+\alpha)} + \sqrt{\left[\frac{\alpha(1+2\alpha)}{\beta_F(1+\alpha)}\right]^2 + \frac{(1+2\alpha)^2}{\beta_F(1+\alpha)}} \quad (25)$$

or when simplified:

$$NF_{OPT,MIN} = 1 + \frac{1+2r_{EF}g_{m,OPT}}{\sqrt{\beta_F(1+r_{EF}g_{m,OPT})}} \quad (26)$$

The form of the obtained expressions again resembles the ID case.

5. AN OVER-ALL EXAMPLE

To prove the validity of the introduced concept a fully realistic example, also tested with the SpectreRF simulation tool, is presented. Here, the performance parameters of both transformer-feedback degenerated and inductively-degenerated LNA are calculated by means of the foregoing expressions, and subsequently compared with respect to each other as well as with respect to the simulation results.

Referring to a 50GHz SiGe technology and frequency of operation f_0=2.4GHz, the following parameters for the optimum-minimum noise-figure point are obtained: C_Π=1.56pF, β_F=105 and current density J_C=0.15x10^9. Dimensions of the chosen transistors are 24x(0.4x5)um^2 for TFD and 20x(0.4x5)um^2 for ID, as we wanted to have the same optimum noise resistance. In case of the ID, the inductors in the emitter and the base of the first transistor are denoted as L_E and L_B, respectively. The power-matched parameters of the TFD LNA are: L_1=1.95nH, L_2=4.7nH, k=0.9 and n=1.4. The power-matched parameters of the ID LNA are L_B=2.2nH and L_E=0.36nH. In both cases a 1nH bond-wire inductance is placed at the input of the LNA. The other parameters are shown in Tabs. 1, 2, 3 and 4.

$g_{m,OPT}$[mS]	f_T[GHz]	PG[dB]	$R_{S,OPT}$[Ω]	NF_{OPT}[dB]
0.3	29.6	15.8	59.3	1

Table 1 Calculated parameters of the power-matched ID LNA.

$g_{m,OPT}$[S]	f_T[GHz]	PG[dB]	$R_{S,OPT}$[Ω]	NF_{OPT}[dB]	$IIP3$[dBm]
0.27	26	16	60	1.2	1.5

Table 2 Simulated parameters of the power-matched ID LNA.

$g_{m,OPT}$[S]	f_T[GHz]	PG[dB]	$R_{S,OPT}$[Ω]	NF_{OPT}[dB]
0.3	28.5	15.6	59.8	1.12

Table 3 Calculated parameters of the power-matched TFD LNA

$g_{m,OPT}$[S]	f_T[GHz]	PG[dB]	$R_{S,OPT}$[Ω]	NF_{OPT}[dB]	$IIP3$[dBm]
0.29	24	16	60	1.25	1

Table 4 Simulated parameters of the power-matched TFD LNA.

As apparent from Tabs. 1-4, the noise figure, the power gain and the linearity of the TFD LNA are not lagging their ID counterparts. What is more, owing to, at the same time, more versatile matching capabilities, the TFD overwhelms traditionally the "best" ID.

Finally, the exemplified accuracy of all the derived expressions has qualified them to be considered as a good estimator for the performance parameters of the discussed LNA's. Also, it is possible to find out quickly whether the chosen amplifier parameters satisfy the required specifications and accordingly speed up the complete design process.

6. CONCLUSIONS

The tendency of moving to higher transition frequencies, lower supply voltages and smaller dimensions must be directly followed by certain changes in the design philosophy, as well.

This has resulted in transformer-feedback degenerated low-noise amplifier, presented in this paper. It has been shown that controlling the transformer coupling coefficient, the power match becomes possible, contrary to inductive-degeneration, even for the highest values of f_T, with at the same time larger and design-safe primary inductor of the implemented transformer.

Also, a detailed analysis, resulting in tractable expressions for the amplifier performance parameters, being power gain and noise figure, is presented. These expressions provide one with a very good tool for a quick estimation whether the required specifications are satisfied and serve as a good initial guess for the subsequent simulations.

Finally, comparing the performances of the introduced topology with the inductively degenerated one, it can be noticed that obtained results are rather similar, implying that due to superior matching scheme, transformer-feedback topology can actually become the favorite one.

7. REFERENCES

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